$(e^x)' = e^x$; $(\ln x)' = \frac{1}{x}, \forall x>0$; $(a^{f(x)})' = (\ln a) \cdot f'(x) \cdot a^{f(x)}$

$(e^{f(x)})' = f'(x) e^{f(x)}$; $(\ln f(x))' = \frac{f'(x)}{f(x)}$; $(\ln ab) = \ln a + \ln b$; $(\ln \frac{a}{b}) = \ln a - \ln b$

$\ln x^a = a \ln x$; $\log_a b = \frac{\ln b}{\ln a}$

PRINTABLE VERSION

Sol

Quiz 16

Question 1

Differentiate: $y = e^{3x^2 - 2}$

$y' = (3x^2-2)' e^{3x^2-2}$
$= (6x) e^{3x^2-2}$

a) $y' = 3e^{3x^2-2}$

b) $y' = 6e^{3x^2-2}$

c) $y' = 3xe^{3x^2-2}$

d) $y' = 6xe^{3x^2-2}$

e) $y' = e^{3x^2-2}$

Question 2

Differentiate: $y = \underline{3x}\underline{e^{4x^2}}$

By Product Rule

$y' = (3x)' e^{4x^2} + 3x (e^{4x^2})'$
$= 3e^{4x^2} + 3x (4x^2)' e^{4x^2}$
$= 3e^{4x^2} + 24x^2 e^{4x^2}$

a) $y' = 3e^{4x^2}$

b) $y' = 3e^{4x^2} - 24x^2e^{4x^2}$

c) $y' = 3e^{8x}$

d) $y' = 3e^{4x^2} - 3xe^{4x^2}$

e) $y' = e^{4x^2} + 8x^2e^{4x^2}$

Question 3

Differentiate: $y = \cos(3e^{4x})$

By Chain Rule.

$y' = -\sin(3e^{4x}) \cdot [(3e^{4x})']$
$= -12e^{4x} \cdot \sin(3e^{4x})$

a) $y' = -3e^{4x}\sin(3e^{4x})$

b) $y' = -12e^{4x}\cos(3e^{4x})\sin(3e^{4x})$

c) $y' = 12e^{4x}\sin(3e^{4x})$

d) $y' = 12e^{4x}\cos(3e^{4x})\sin(3e^{4x})$

e) $y' = -12e^{4x}\sin(3e^{4x})$

Question 4

Differentiate: $y = 3e^{\sqrt{5x}}$

$y' = 3 \cdot \frac{1}{2} \frac{5}{\sqrt{5x}} e^{\sqrt{5x}}$
$= \frac{15}{2} \frac{1}{\sqrt{5x}} e^{\sqrt{5x}}$

a) $y' = \frac{15}{\sqrt{5x}} e^{\sqrt{5x}}$

b) $y' = \frac{15}{2\sqrt{5x}} e^{\sqrt{5x}}$

c) $y' = \frac{3}{\sqrt{5x}} e^{\sqrt{5x}}$

d) $y' = \frac{15}{2}\sqrt{5x}\, e^{\sqrt{5x}}$

e) $y' = 15\sqrt{5x}\, e^{\sqrt{5x}}$

Q5,

$y' = \sec^2(7^{5x^2}) \cdot [(7^{5x^2})']$
$= (\ln 7)(10x)(7^{5x^2}) \cdot \sec^2(7^{5x^2})$

Question 5

Differentiate: $y = \tan(7^{5x^2})$

a) $y' = 7^{5x^2}(10x)\sec^2\left(7^{5x^2}\right)$

b) $y' = 7^{5x^2}\ln(7)(10x)\sec^2\left(7^{5x^2}\right)$

c) $y' = 7^{5x^2}\ln(7)(10x)\tan\left(7^{5x^2}\right)$

d) $y' = 7^{5x^2}\ln(7)(10x)\sec\left(7^{5x^2}\right)\tan\left(7^{5x^2}\right)$

e) $y' = 7^{5x^2}(10x)\tan\left(7^{5x^2}\right)$

Question 6

Differentiate: $y = \ln(3x^2 + 4)$

$$y' = \frac{(3x^2+4)'}{3x^2+4} = \frac{6x}{3x^2+4}.$$

a) $y' = -\frac{1}{(3x^2+4)^2}$

b) $y' = \frac{3}{3x^2+4}$

c) $y' = -\frac{6x}{(3x^2+4)^2}$

d) $y' = \frac{1}{3x^2+4}$

e) $y' = \frac{6x}{3x^2+4}$

Question 7

Differentiate $y = e^{3x}\ln(x^2)$

Q7. By product Rule,

$$y' = 3e^{3x}\ln(x^2) + e^{3x}\cdot\frac{2}{x}$$
$$= 3e^{3x}\cdot 2\ln x + \frac{2e^{3x}}{x}$$
$$= 6e^{3x}\ln x + \frac{2e^{3x}}{x}$$

a) $y' = e^{3x}\ln(2x) - \frac{e^x}{x}$

b) $y' = 6e^{3x}\ln(x) + \frac{2e^{3x}}{x}$

c) $y' = 3e^{3x}\ln(2x) - \frac{e^{3x}}{x}$

d) $y' = \ln\left(x^2\right) + \frac{e^{3x}}{x}$

e) $y' = e^{3x}\ln\left(x^2\right) + 3e^x$

Question 8

Determine the domain and differentiate $f(x) = \ln\sqrt{9 + 4x^2}$

Q8. Domain of f:

$$D(f) = \{\sqrt{9+4x^2} > 0\} = \{9+4x^2 > 0\} = \{x \in \mathbb{R}\} = (-\infty, \infty)$$

$$f'(x) = \frac{(\sqrt{9+4x^2})'}{\sqrt{9+4x^2}} = \frac{\frac{1}{2}\frac{8x}{\sqrt{9+4x^2}}}{\sqrt{9+4x^2}} = \frac{4x}{9+4x^2}$$

a) $\mathrm{dom}(f) = (0, \infty);\ f'(x) = \frac{4x}{4x^2+9}$

b) $\mathrm{dom}(f) = (0, \infty);\ f'(x) = \frac{1}{4x^2+9}$

c) $\mathrm{dom}(f) = (-\infty, \infty);\ f'(x) = \frac{4x}{4x^2+9}$

d) $\mathrm{dom}(f) = (-\infty, \infty);\ f'(x) = \frac{1}{4x^2+9}$

e) $\mathrm{dom}(f) = (-\infty, \infty);\ f'(x) = \frac{1}{8x^2+18}$

Question 9

Calculate the derivative by logarithmic differentiation:

9. $\ln g(x) = \ln x^5 + \ln(x-1)^2 - \ln(x+2)^2 - \ln(x^2+1)^4$
$= 5\ln x + 2\ln(x-1) - 2\ln(x+2) - 4\ln(x^2+1)$

$\Rightarrow$ Do derivative on both sides:
$$\frac{g'(x)}{g(x)} = \frac{5}{x} + \frac{2}{x-1} - \frac{2}{x+2} - \frac{4\cdot(2x)}{x^2+1}$$

$$g(x) = \frac{x^5(x-1)^2}{(x+2)^2(x^2+1)^4}.$$

$$\Rightarrow g'(x) = g(x)\left[\frac{5}{x} + \frac{2}{x-1} - \frac{2}{x+1} - \frac{8x}{x^2+1}\right]$$

a) $g'(x) = \frac{x^5(x-1)^2}{(x-2)^2(x^2+1)^4}\left(\frac{5}{x} - \frac{2}{x-1} + \frac{2}{x+2} + \frac{8x}{x^2-1}\right)$

b) $g'(x) = \frac{x^5(x-1)^2}{(x+2)^2(x^2-1)^4}\left(\frac{5}{x^2} - \frac{2}{(x-1)^2} - \frac{2}{x+2} - \frac{8x}{x^2+1}\right)$

c) $g'(x) = \frac{x^5(x-1)^2}{(x+2)^2(x^2+1)^4}\left(\frac{1}{x} - \frac{1}{x-1} - \frac{1}{x+2} - \frac{4}{x^2+1}\right)$

d) $g'(x) = \frac{x^5(x-1)^2}{(x+2)^2(x^2-1)^4}\left(\frac{5}{x} + \frac{2}{x-1} - \frac{2}{x+2} - \frac{8x}{x^2+1}\right)$

e) $g'(x) = \frac{x^5(x-1)^2}{(x+2)^2(x^2+1)^4}\left(\frac{5}{x} - \frac{2}{x-1} - \frac{2}{x+2} - \frac{4}{x^2+1}\right)$

Question 10

Find the points of inflection for the function: $f(x) = x^2\ln(x/2)$

$= x^2(\ln x - \ln 2)$
$= x^2\ln x - x^2\ln 2$

a) $\left(2e^{-3/2}, 0\right)$

b) $\left(2e^{-3/2}, -6e^{-3}\right)$

c) $\left(2e^{3/2}, 0\right)$

d) $\left(\frac{1}{2}e^{-3/2}, -6e^{-3}\right)$

$f'(x) = 2x\ln x + x^2\cdot\frac{1}{x} - 2x\cdot\ln 2$

$f''(x) = 2\ln x + \frac{2x}{x} + 1 - 2\ln 2$

$= 2\ln x + 3 - 2\ln 2$

$\Rightarrow f''(x) = 0 \Rightarrow \ln x = -\frac{3}{2} + \ln 2 \Rightarrow x = e^{-\frac{3}{2}+\ln 2} = e^{-\frac{3}{2}}\cdot e^{\ln 2} = 2e^{-\frac{3}{2}} \Rightarrow (2e^{-\frac{3}{2}}, -6e^{-3})$

and $f(2e^{-\frac{3}{2}}) = 4e^{-3}\cdot\ln\left(\frac{2e^{-\frac{3}{2}}}{2}\right) = 4e^{-3}\cdot(\ln e^{-\frac{3}{2}}) = -\frac{3}{2}\cdot 4e^{-3} = -6e^{-3}$

e) $\left(2e^{3/2}, -6e^3\right)$

Question 11

Differentiate the given function: $f(x) = \frac{\log_9 x}{x^4}$ $= \frac{\frac{\ln x}{\ln 9}}{x^4}$.

a) $f'(x) = \frac{1}{x^6\ln(9)} - \frac{4\ln(x)}{x^6\ln(9)}$

b) $f'(x) = \frac{1}{x^4(\ln(9))^2} - \frac{4\ln(x)}{x^4\ln(9)}$

c) $f'(x) = \frac{1}{x^5\ln(9)} - \frac{4\ln(x)}{x^5\ln(9)}$

d) $f'(x) = \frac{1}{x^5(\ln(9))^2} - \frac{4\ln(x)}{x^5\ln(9)}$

e) $f'(x) = \frac{1}{x^4\ln(9)} - \frac{4\ln(x)}{x^4\ln(9)}$

$f(x) = \frac{1}{\ln 9}\left(\frac{\ln x}{x^4}\right).$

$f'(x) = \frac{1}{\ln 9}\left[\frac{\frac{x^4}{x} - 4x^3\ln x}{x^8}\right]$

$= \frac{1}{\ln 9}\left[\frac{x^3 - 4x^3\ln x}{x^8}\right]$

$= \frac{1}{\ln 9}\left[\frac{1-4\ln x}{x^5}\right]$