# PRINTABLE VERSION

## Quiz 15

Sol

### Question 1

Determine whether or not the given function is one-to-one and, if so, find the inverse. If $f(x) = 6x - 5$ has an inverse, give the domain of $f^{-1}$.

$f'(x) = 6 > 0 \Rightarrow f$ is 1-1

① Let $y = f(x) = 6x - 5$

② Switch $x$ & $y \Rightarrow x = 6y - 5$

③ Solve $y$. $y = \frac{x+5}{6}$

$f^{-1} = \frac{x+5}{6}$

$D(f^{-1}) = \mathbb{R}$

a) Not one-to-one.

b) $f^{-1}(x) = \frac{1}{6}x + \frac{5}{6}$; domain: $(-\infty, \infty)$

c) $f^{-1}(x) = -\frac{1}{6}x - \frac{5}{6}$; domain: $(-5, \infty)$

d) $f^{-1}(x) = 6x - 5$; domain: $(-\infty, \infty)$

e) $f^{-1}(x) = 6x + 5$; domain: $(-\infty, -5)$

### Question 2

Determine whether or not the given function is one-to-one and, if so, find the inverse. If $f(x) = x^2 - 4x + 1$ has an inverse, give the domain of $f^{-1}$.

$f'(x) = 2x - 4 \Rightarrow f'(x) > 0$ as $x > \frac{1}{2}$ and $f'(x) < 0$ as $x < \frac{1}{2}$.

$\Rightarrow f$ is NOT monotone

$\Rightarrow f$ is NOT 1-1.

a) $f^{-1}(x) = \sqrt{x+3} + 2$; domain: $(-3, \infty)$

b) Not one-to-one.

c) $f^{-1}(x) = \sqrt{x} - \frac{1}{4}$; domain: $\left(\frac{1}{4}, \infty\right)$

d) $f^{-1}(x) = -\sqrt{x+3} - 2$; domain: $(-3, \infty)$





e) $f^{-1}(x) = \sqrt{x} - \frac{1}{4}$; domain: $(-\infty, 0)$

### Question 3

Determine whether or not the given function is one-to-one and, if so, find the inverse. If $f(x) = (-3x - 5)^4$ has an inverse, give the domain of $f^{-1}$.

3. $f'(x) = 4(-3x+5)^3 \cdot (-3x+5)'$
$= -12(-3x+5)^3$

and $f'(x) > 0$ as $x > \frac{5}{3}$, $f'(x) < 0$ as $x < \frac{5}{3}$

$\Rightarrow f$ is NOT monotone $\Rightarrow f$ is NOT 1-1.

a) $f^{-1}(x) = \frac{x^{1/4} - 5}{3}$; domain: $(-\infty, \infty)$

b) Not one-to-one

c) $f^{-1}(x) = (5 + 3x)^{1/4}$; domain: $\left(-\infty, \frac{5}{3}\right)$

d) $f^{-1}(x) = \frac{5 - x^{1/4}}{3}$; domain: $\left(\frac{5}{3}, \infty\right)$

e) $f^{-1}(x) = \frac{5 - x^{1/4}}{3}$; domain: $(0, \infty)$

### Question 4

Determine whether or not the given function is one-to-one and, if so, find the inverse. If $f(x) = (1 - 4x^4)^3$ has an inverse, give the domain of $f^{-1}$.

4. $f'(x) = 3(1-4x^4)^2(1-4x^4)'$
$= 3(-16x^3)(1-4x^4)^2$
$= -48x^3(1-4x^4)^2$.

$\Rightarrow f'(x) > 0$ as $x < 0$ & $f'(x) < 0$ as $x > 0$

$\Rightarrow f$ is NOT monotone

$\Rightarrow f$ is NOT 1-1.

a) $f^{-1}(x) = \left(-\frac{1}{4}x^{1/3} + \frac{1}{4}\right)^{1/4}$; domain: $(0, \infty)$

b) Not one-to-one

c) $f^{-1}(x) = (1 - 4x^4)^{1/3}$; domain: $(-\infty, \infty)$

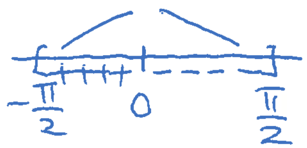
d) $f^{-1}(x) = (1 - 4x^4)^{1/3}$; domain: $(0, )$

e) $f^{-1}(x) = \left(-\frac{1}{4}x^{1/3} + \frac{1}{4}\right)^{1/4}$ ; domain: $(-\infty, \infty)$

**Question 5**

Determine whether or not the given function is one-to-one and, if so, find the inverse: $f(x) = \frac{3}{2}\cos(x)$ with $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$f'(x) = -\frac{3}{2}\sin(x)$




a) $f^{-1}(x) = \arccos\left(\frac{2}{3}x\right)$

b) $f^{-1}(x) = \frac{3}{2}\sec(x)$

c) Not one-to-one

$\Rightarrow$ NoT monotone

d) $f^{-1}(x) = \frac{3}{2}\sin(x)$

$\Rightarrow$ NoT 1-1,

e) $f^{-1}(x) = \sec\left(\frac{2}{3}x\right)$

**Question 6**

Given the following function, with $k$ as a constant, find the values of $k$ for which $f(x) = \frac{1}{3}x^3 - 2x^2 + kx$ is one-to-one.

$D(f) = \mathbb{R}$

$f'(x) = x^2 - 4x + k$

a) $k \geq -4$

$= (x^2 - 4x + 4) - 4 + k$

b) $k \leq \frac{1}{4}$

$= (x-2)^2 - 4 + k$

c) $k \geq 4$

$\Rightarrow f'(x) > 0$ on $\mathbb{R}$ if $-4 + k \geq 0$

$\Rightarrow k \geq 4$





d) $-\frac{1}{4} \leq k \leq \frac{1}{4}$

e) $-4 \leq k \leq 4$

**Question 7**

Suppose that $f$ has an inverse and $f(4) = -1$, $f'(4) = \frac{4}{11}$. What is $(f^{-1})'(-1)$?

$\underset{a}{4}$, $\underset{b}{-1}$ $\Rightarrow f^{-1}(-1) = 4$.

$(f^{-1})'(-1) = \frac{1}{f'(4)} = \frac{1}{\frac{4}{11}} = \frac{11}{4}$

a) $\frac{11}{2}$

b) $-\frac{4}{11}$

c) $\frac{4}{11}$

d) $\frac{11}{4}$

e) $\frac{15}{4}$

8. $f^{-1}(-5) = 1$, $f'(x) = -3x^2 - 3$ and $f'(1) = -3 - 3 = -6$.

Then $(f^{-1})'(-5) = \frac{1}{f'(1)} = \frac{-1}{6}$

**Question 8**

Suppose that $f(x) = -x^3 - 3x - 1$ is differentiable and has an inverse and $f(1) = -5$. Find $(f^{-1})'(-5)$.

$\underset{a}{1}$, $\underset{b}{-5}$

a) $-\frac{1}{12}$

b) $-\frac{1}{6}$

c) $-\frac{1}{3}$

d) $\frac{1}{5}$

e) $\frac{1}{3}$

**Question 9**

Suppose that $f(x) = 8x - 3\cos x$ is differentiable and has an inverse and $f\left(\frac{\pi}{2}\right) = 4\pi$. Find $\left(f^{-1}\right)'(4\pi)$.

$a$ $b$

$\Rightarrow f^{-1}(4\pi) = \frac{\pi}{2}$, $f'(x) = 8 - 3\sin x$ & $f'\left(\frac{\pi}{2}\right) = 8 - 3 = 5$.

$\Rightarrow (f^{-1})'(4\pi) = \frac{1}{f'\left(\frac{\pi}{2}\right)} = \frac{1}{5}$

a) $-\frac{1}{5}$

b) $\frac{1}{10}$

c) $\frac{2}{5}$

d) $\frac{1}{5}$

e) $-\frac{2}{5}$

10. $f(3) = 2 \Rightarrow f^{-1}(2) = 3$, & $f'(x) = \frac{x-1-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$ and $f'(3) = \frac{-2}{4} = -\frac{1}{2}$

$\Rightarrow (f^{-1})'(2) = \frac{1}{f'(3)} = \frac{1}{-\frac{1}{2}} = -2$

**Question 10**

Suppose that $f(x) = \frac{x+1}{x-1}$ is differentiable and has an inverse for $x > 1$ and $f(3) = 2$. Find $\left(f^{-1}\right)'(2)$.

$a$ $b$

a) 2





b) $-2$

c) $-4$

d) $-1$

e) 4