① Vertical asymptote : $x=c$ if $f(x)\to\pm\infty$ as $x\to c^{+}$ or $x\to c^{-}$

② horizontal asymptote: $y=L$ if $\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$

PRINTABLE VERSION

Quiz 14

Question 1

Find the vertical and horizontal asymptotes of $f(x)=\dfrac{x}{x^2-1}$.

① $f(x)\to\pm\infty$ if $x^2-1=0$
$\Rightarrow (x+1)(x-1)=0$
$\Rightarrow x=\pm1$. (Vertical.)

② $\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$
$\Rightarrow y=0$ (horizontal)

a) vertical asymptote: $x=\pm1$; horizontal asymptote: $y=1$.

b) vertical asymptote: $x=\pm1$; horizontal asymptote: $y=0$.

c) vertical asymptote: $x=\pm1$; no horizontal asymptote.

d) vertical asymptote: $x=0$; horizontal asymptote: $y=\pm1$.

e) no vertical asymptote; horizontal asymptote: $y=6$.

$\dfrac{3\sqrt{x}}{(\sqrt{x}-6)(\sqrt{x}-6)}$

Question 2

Find the vertical and horizontal asymptotes of $f(x)=\dfrac{3\sqrt{x}}{x-12\sqrt{x}+36}$.

① $f(x)\to\pm\infty$ if $(\sqrt{x}-6)^2=0$
$\Rightarrow x=36$
(Vertical.)

② $\lim_{x\to\infty} f(x)=0$
$\lim_{x\to-\infty} f(x)=0$
$\Rightarrow y=0$ (horizontal)

a) vertical asymptote: $x=0$; no horizontal asymptote.

b) vertical asymptote: $x=\pm36$; horizontal asymptote: $y=3$.

c) vertical asymptote: $x=36$; horizontal asymptote: $y=0$.

d) no vertical asymptote; horizontal asymptote: $y=\pm\frac{1}{2}$.

e) vertical asymptote: $x=0$; horizontal asymptote: $y=\frac{1}{2}$.

Question 3

Find the vertical and horizontal asymptotes of $f(x)=\dfrac{6\sin(x)+3}{\sin(x)+1}$.

3. ① $f(x)\to\pm\infty$ if $\sin(x)+1=0$
$\Rightarrow \sin(x)=-1$
$x=\frac{3\pi}{2}+2n\pi$
(Vertical)

② $\lim_{x\to\infty} f(x)=$ DNE

a) vertical asymptote: $x=0$; horizontal asymptote: $y=\pm1$.

b) vertical asymptote: $x=\dfrac{3\pi}{2}+2\pi n$; horizontal asymptote: $y=1$.

c) vertical asymptote: $x=\dfrac{3\pi}{2}+2\pi n$; no horizontal asymptote.

d) no vertical asymptote; horizontal asymptote: $y=\pm1$.

e) vertical asymptote: $x=\dfrac{3\pi}{2}+2\pi n$; horizontal asymptote: $y=0$.

Question 4

Determine whether or not the graph of $f(x)=2(x-4)^{4/5}$ has a vertical tangent or vertical cusp at $x=4$.

Check $f'(x)$ at $x\to4$ if
$\Rightarrow \lim_{x\to4} f'(x)=\infty$ or $\lim_{x\to4} f'(x)=-\infty$
$\Rightarrow \lim_{x\to4^-} f'(x)=\pm\infty$ and $\lim_{x\to4^+} f'(x)=\mp\infty$

So, $f'(x)=2\cdot\frac{4}{5}\cdot\frac{1}{(x-4)^{1/5}}$ and
$\lim_{x\to4^-} f'(x)=-\infty$, $\lim_{x\to4^+} f'(x)=\infty$ $\Rightarrow$ Vertical cusp.

a) vertical tangent

b) vertical cusp

c) both

d) neither

Question 5

Determine whether or not the graph of $f(x)=9x^{3/5}-7x^{6/5}$ has a vertical

5. $f(x)=9x^{\frac{3}{5}}-7x^{\frac{6}{5}}$. $f'(x)=\frac{27}{5}x^{-\frac{2}{5}}-\frac{42}{5}x^{\frac{1}{5}}$

$$=\frac{27-42x^{\frac{3}{5}}}{5x^{\frac{2}{5}}}$$

tangent or vertical cusp at $x=0$. $\lim_{x\to 0^+} f'(x)=\infty$, $\lim_{x\to 0^-} f'(x)=\infty$

$\Rightarrow$ Vertical tangent

a) neither

b) vertical cusp

c) both

d) vertical tangent

6. $f'(x)=+\frac{3}{7}(6-x)^{-\frac{4}{7}}=\frac{3}{7}\cdot\frac{1}{(6-x)^{\frac{4}{7}}}$

Question 6

Determine whether or not the graph of $f(x)=8-(6-x)^{3/7}$ has a vertical tangent or vertical cusp at $x=6$.

$\lim_{x\to 6^+} f'(x)=\infty$, $\lim_{x\to 6^-} f'(x)=\infty$

$\Rightarrow$ Vertical tangent.

a) vertical cusp

b) both

c) neither

d) vertical tangent

$f'(x)=9(x-8)^{\frac{1}{3}}+3x\cdot\frac{1}{(x-8)^{\frac{2}{3}}}=\frac{9(x-8)+3x}{(x-8)^{\frac{2}{3}}}$

$=\frac{12x-72}{(x-8)^{\frac{2}{3}}}$

Question 7

Determine whether or not the graph of $f(x)=9x\sqrt[3]{x-8}$ has a vertical tangent or vertical cusp at $x=8$.

$\lim_{x\to 8^-} f'(x)=\infty$, $\lim_{x\to 8^+} f'(x)=\infty$

$\Rightarrow$ Vertical tangent,

a) vertical cusp

b) vertical tangent

c) neither

8. $D(f)=\{x\neq 0\}$, $f'(x)=16x-\frac{2}{x^2}$; $f''(x)=16+4\frac{1}{x^3}$

Critical point : $f'(x)=0 \Rightarrow 16x=\frac{2}{x^2} \Rightarrow x^3=\frac{2}{16} \Rightarrow x=\pm\frac{1}{2}$

point of Inflection : ~~none~~ $f''(x)=0 \Rightarrow x=-\frac{1}{\sqrt[3]{4}}$

Number line of f' — Number line of f''

d) both

$-\frac{1}{2}$ 0 $\frac{1}{2}$ — up down up, $-\frac{1}{\sqrt[3]{4}}$ 0

Question 8

Which of the following is true about the graph of $f(x)=8x^2+\frac{2}{x}-4$?

a) $f(x)$ is increasing on the interval $(-\infty,0)$. X $\Rightarrow (\frac{1}{2},\infty)$

b) $f(x)$ has a vertical asymptote at $x=2$. X $\Rightarrow$ vertical cusp at $x=0$

c) $f(x)$ is concave down on the interval $(0,\infty)$. X $\Rightarrow (-\frac{1}{\sqrt[3]{4}},0)$

d) $f(x)$ has a point of inflection at the point $(0,-4)$. X $\Rightarrow (-\frac{1}{\sqrt[3]{4}}, f(-\frac{1}{\sqrt[3]{4}}))$

e) $f(x)$ has a local minimum at the point $\left(\frac{1}{2},2\right)$. ✓

Question 9 $f'(x)=1+2\cos(2x)$; $f''(x)=-4\sin(2x)$.

Which of the following is true about the graph of $f(x)=x+\sin(2x)+4$ on the interval $[0,\pi]$?

critical point : $f'(x)=0 \Rightarrow \cos(2x)=-\frac{1}{2}$

$\Rightarrow 2x=\frac{2\pi}{3},\frac{4\pi}{3} \Rightarrow x=\frac{\pi}{3},\frac{2\pi}{3}$

a) $f(x)$ has a point of inflection at the point $(0,4)$. X

b) $f(x)$ is decreasing on the interval $(0,1\pi)$. X $(\frac{\pi}{3},\frac{2\pi}{3})$

c) $f(x)$ has a local maximum at the point $\left(\frac{\pi}{3},\frac{\pi}{3}+\frac{\sqrt{3}}{2}+4\right)$. ✓

d) $f(x)$ is concave up on the interval $\left(0,\frac{2\pi}{3}\right)$. X $(\frac{\pi}{2},\pi)$.

point of Inflection, $f''(x)=0 \Rightarrow \sin(2x)=0$

$2x=0,\pi,2\pi \Rightarrow x=0,\frac{\pi}{2},\pi$

Number line of f' — Number line of f''

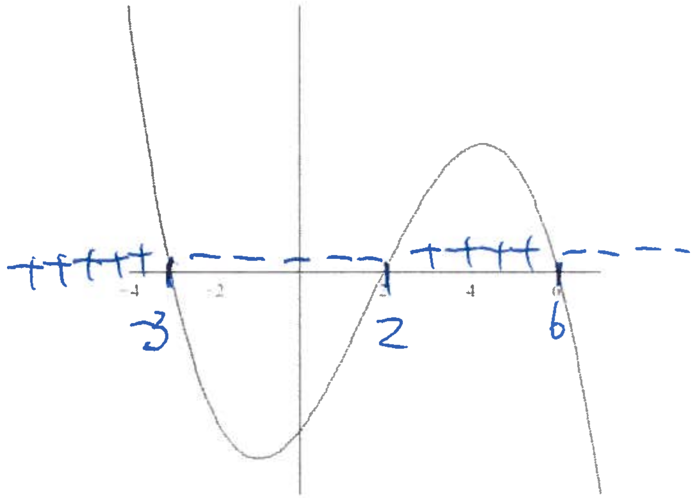
concave down up

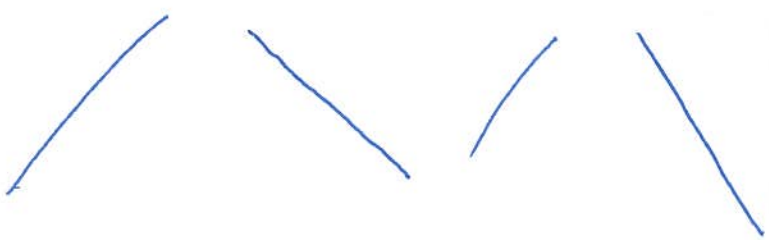
0 $\frac{\pi}{3}$ $\frac{2\pi}{3}$ π — 0 $\frac{\pi}{2}$ π

e) $f(x)$ is increasing on the interval $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$. X $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$

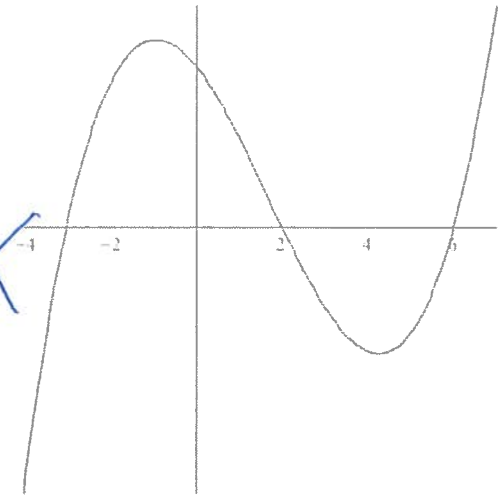
Question 10

The graph of $f'(x)$ is shown below. Which of the following could represent the graph of $f(x)$?

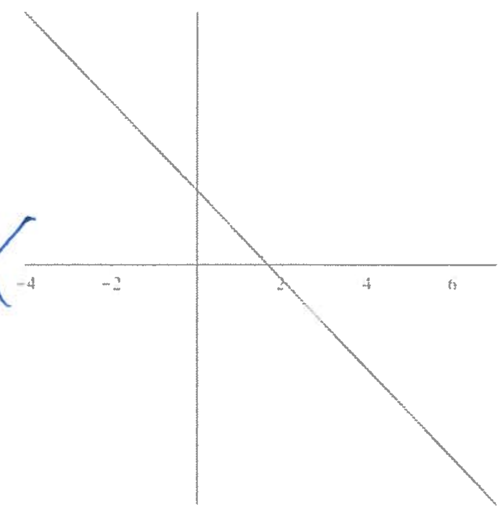




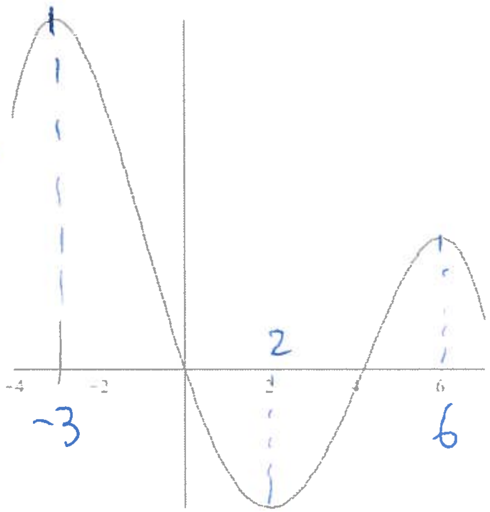
a) X



b) X



c)



d)