PRINTABLE VERSION

Quiz 13

Sol

Question 1

Describe the concavity of the graph of $f(x) = x^3 - 2x + 6$ and find the points of inflection (if any).

$f'(x) = 3x^2 - 2$; $f''(x) = 6x$, $f''(x) = 0 \Rightarrow x = 0$; $f''(x)$ DNE: NONE

Number line of f''

f'': $- - - - \ \underset{0}{|} \ + + + + +$

a) concave down on $\left(-\infty, \frac{1}{3}\right)$; concave up on $\left(\frac{1}{3}, \infty\right)$; pt of inflection $\left(\frac{1}{3}, 0\right)$.

concave down $(f''<0)$: $(-\infty, 0)$

concave up $(f''>0)$: $(0, \infty)$

b) concave down on $(-\infty, \infty)$; no points of inflection

pt of inflection $(0, f(0)) = (0, 6)$.

c) concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; pt of inflection $(0, 6)$.

d) concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; pt of inflection $(0, 6)$.

e) concave up on $(-\infty, \infty)$; no points of inflection

Question 2

$f'(x) = 7x^3 - 7x$; $f''(x) = 21x^2 - 7 = 7(3x^2 - 1) = 21\left(x^2 - \frac{1}{3}\right) = 21\left(x + \frac{\sqrt{3}}{3}\right)\left(x - \frac{\sqrt{3}}{3}\right) = 0$

Describe the concavity of the graph of $f(x) = \frac{7}{4}x^4 - \frac{7}{2}x^2$ and find the points of inflection (if any).

$x = -\frac{\sqrt{3}}{3}$ or $\frac{\sqrt{3}}{3}$

Number line of f''

$f''(x)$ DNE: NONE

$+ + + + \ \underset{-\frac{\sqrt{3}}{3}}{|} \ - - - \ \underset{\frac{\sqrt{3}}{3}}{|} \ + + + +$

a) concave down on $(-\infty, \infty)$; no points of inflection

b) concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; pt of inflection $(0, 0)$.

concave up $(f''>0)$: $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

concave down $(f''<0)$: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

points of Inflection: $\left(\frac{\sqrt{3}}{3}, f\left(\frac{\sqrt{3}}{3}\right)\right) = \left(\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$

$\left(-\frac{\sqrt{3}}{3}, f\left(-\frac{\sqrt{3}}{3}\right)\right) = \left(-\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$.

c) concave up on $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$; concave down on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$; pts of inflection $\left(\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$ and $\left(-\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$.

d) concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; pt of inflection $(0, 0)$.

e) concave down on $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$; concave up on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$; pts of inflection $\left(\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$ and $\left(-\frac{\sqrt{3}}{3}, -\frac{35}{36}\right)$.

Question 3

$9x^2 - 4 = 0 \Rightarrow 9\left(x^2 - \frac{4}{9}\right) = 0 \Rightarrow 9\left(x - \frac{2}{3}\right)\left(x + \frac{2}{3}\right) = 0$

$\Rightarrow D(f) = \{x \neq \pm\frac{2}{3}\}$

Describe the concavity of the graph of $f(x) = \frac{2x}{9x^2 - 4}$ and find the points of inflection (if any).

$f'(x) = \frac{-2(9x^2+4)}{(9x^2-4)^2}$; $f''(x) = \frac{108x(3x^2+4)}{(9x^2-4)^3}$, $f''(x) = 0 \Rightarrow x = 0$.

a) concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; pt of inflection $(0, 0)$.

$f''(x)$ DNE $\Rightarrow x = \cancel{\pm}\frac{2}{3}$ (NOT IN DOMAIN)

b) concave down on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(0, \frac{2}{3}\right)$; concave up on $\left(-\frac{2}{3}, 0\right)$ and $\left(\frac{2}{3}, \infty\right)$; pt of inflection $(0, 0)$.

c) concave down on $\left(-\infty, \frac{2}{3}\right)$; concave up on $\left(\frac{2}{3}, \infty\right)$; pt of inflection $\left(\frac{2}{3}, 0\right)$.

Number line of f''

$- - - \ \underset{-\frac{2}{3}}{\circ} \ + + + \ \underset{0}{|} \ - - - \ \underset{\frac{2}{3}}{\circ} \ + + + + +$

concave up $(f''>0)$: $\left(-\frac{2}{3}, 0\right) \cup \left(\frac{2}{3}, \infty\right)$

concave down $(f''<0)$: $\left(-\infty, -\frac{2}{3}\right) \cup \left(0, \frac{2}{3}\right)$

point of inflection $(0, f(0)) = (0, 0)$.

4. $f(x)=2(x-3)^{\frac{5}{3}}$. $D(f)=\mathbb{R}$. $f'(x)=2\cdot\frac{5}{3}(x-3)^{\frac{2}{3}}$

$f''(x)=\frac{20}{9}(x-3)^{-\frac{1}{3}}=\frac{20}{9}\frac{1}{\sqrt[3]{x-3}}$ $\quad - - - - - \overset{\circ}{3} + + + + + +$

$f''(x)=0$: NONE; $f''(x)$ DNE: $x=3$

d) concave down on $(-\infty,\infty)$; no points of inflection

e) concave up on $\left(-\frac{2}{3},\frac{2}{3}\right)$; concave down on $\left(-\infty,-\frac{2}{3}\right)$ and $\left(\frac{2}{3},\infty\right)$; pts of inflection $\left(-\frac{2}{3},0\right)$ and $\left(\frac{2}{3},0\right)$.

Question 4

Concave up ($f''(x)>0$): $(3,\infty)$

Concave down ($f''(x)<0$): $(-\infty,+3)$

point of Inflection

$(3,f(3))$

$=(3,0)$

Describe the concavity of the graph of $f(x)=2(x-3)^{5/3}$ and find the points of inflection (if any).

a) concave up on $(-\infty,0))$; concave down on $(0,\infty)$; pt of inflection $(0,0.$

b) concave down on $(-\infty,\infty)$; no points of inflection

c) concave up on $(-\infty,\infty)$; no points of inflection

d) concave down on $(-\infty,3)$; concave up on $(3,\infty)$; pt of inflection $(3,0)$.

e) concave up on $(-\infty,-3)$; concave down on $(-3,\infty)$; pt of inflection $(-3,0)$.

Question 5

$f'(x)=16x-16\cos(2x)$. $f''(x)=16+32\sin(2x)$

point of inflection

Describe the concavity of the graph of $f(x)=8x^2-8\sin(2x)$ on the interval $[0,\pi]$.

$f''(x)=0$

$\sin(2x)=-\frac{1}{2}$

a) concave up on $\left(0,\frac{\pi}{12}\right)$; concave down on $\left(\frac{\pi}{12},\pi\right)$.

$\Rightarrow 2x=\frac{7\pi}{6}$ or $\frac{11\pi}{6}$

$x=\frac{7\pi}{12}$ or $\frac{11\pi}{12}$.

& $f''(x)$ DNE: NONE.

Numberline of f''

$[\;+ + + + + \;\frac{7\pi}{12}\; - - - \;\frac{11\pi}{12}\; + + \;]$ (0 to π)

Concave up: $(0,\frac{7\pi}{12})\cup(\frac{11\pi}{12},\pi)$

Concave down: $(\frac{7\pi}{12},\frac{11\pi}{12})$.

b) concave up on $\left(0,\frac{7\pi}{12}\right)$; concave down on $\left(\frac{7\pi}{12},\pi\right)$.

c) concave up on $\left(0,\frac{7\pi}{12}\right)$ and on $\left(\frac{11\pi}{12},\pi\right)$; concave down on $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$.

d) concave up on $\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$; concave down on $\left(0,\frac{7\pi}{12}\right)$ and on $\left(\frac{11\pi}{12},\pi\right)$.

e) concave down on $(0,\pi)$.

Question 6

$D(f)=\{x\neq 0\}$.

Find c so that the graph of $f(x)=cx^2+x^{-2}$ has a point of inflection at $(4,f(4))$.

$f''(4)=0$ or $f''(4)$ DNE and 4 is in $D(f)$.

a) $c=-\frac{3}{128}$ $\quad f'(x)=2cx-2x^{-3}$

b) $c=-\frac{3}{256}$ $\quad f''(x)=2c+6x^{-4}=2c+\frac{6}{x^4}$

c) $c=\frac{3}{128}$ $\quad \Rightarrow f''(4)=0 \Rightarrow 2c+\frac{6}{4^4}=0$

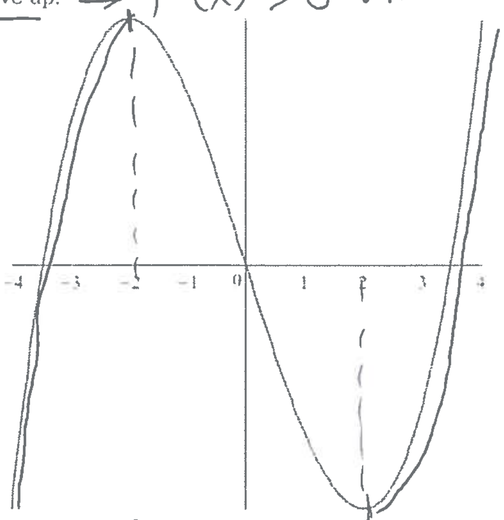
d) $c=\frac{3}{256}$ $\quad c=-\frac{3}{256}$

e) $c=0$

Question 7

The graph of $f'(x)$ is shown below. Give the interval(s) where the graph of

$f(x)$ is concave up. $\Rightarrow f''(x) > 0$ which means f' is increasing



$(-\infty,-2) \cup (2,\infty)$

a) $(-\infty, 0)$ and $(2, \infty)$

b) $(-\infty, -2)$ and $(2, \infty)$

c) $(-\infty, 0)$

d) $(0, \infty)$

e) $(-2, 2)$

$D(f) = \mathbb{R}$.

$f'(x) = 18x^2 - 24x + 6$

Critical pt $= 6(3x^2-4x+1) = 6(3x-1)(x-1)$

$f'(x) = 0 \Rightarrow x = \frac{1}{3}, 1$ — local max ($\frac{1}{3}$), local min (1).

$+++++ \;\; \frac{1}{3} \;\; ----- \;\; 1 \;\; ++++++$

Question 8

Which of the following is true about the graph of $f(x) = 6x^3 - 12x^2 + 6x - 7$?

Point of Inflection: $f''(x) = 36x - 24 = 12(3x-2)$

$f''(x) = 0 \Rightarrow x = \frac{2}{3}$.

concave down | concave up

$- - - - \;\; \frac{2}{3} \;\; + + + + +$

a) $f(x)$ has a local minimum at the point $\left(\frac{1}{3}, -\frac{55}{9}\right)$. X local max

b) $f(x)$ is decreasing on the interval $\left(\frac{1}{3}, 1\right)$. V

c) $f(x)$ has a local maximum at the point $(1, -7)$. X local min

d) $f(x)$ is increasing on the interval $\left(\frac{2}{3}, \infty\right)$. X $(-\infty, \frac{1}{3}) \cup (1, \infty)$.

e) $f(x)$ has a point of inflection at the point $(1, -7)$. X $\left(\frac{2}{3}, f(\frac{2}{3})\right)$

Question 9 $D(f) = \mathbb{R}$. $f'(x) = 6\sin^2(x)\cos(x) + 3\cos(x)$

Which of the following is true about the graph of $f(x) = 2\sin^3(x) + 3\sin(x) + 1$ on the interval $[0, \pi]$? $= 3\cos(x)(2\sin^2(x)+1)$

Critical pt. $f'(x) = 0 \Rightarrow \cos(x) = 0 \Rightarrow x = \frac{\pi}{2}$

X a) $f(x)$ has a local minimum (max) at the point $\left(\frac{\pi}{2}, 6\right)$.

X b) $f(x)$ is increasing on the interval $\cancel{\left(\frac{\pi}{2}, \pi\right)}$. $(0, \frac{\pi}{2})$

$0 \;\; ++++ \;\; \frac{\pi}{2} \;\; ---- \;\; \pi$

X c) $f(x)$ is concave down on the interval $(0, \pi)$.

V d) $f(x)$ has points of inflection at the points $\left(\frac{\pi}{4}, 2\sqrt{2}+1\right)$ and $\left(\frac{3\pi}{4}, 2\sqrt{2}+1\right)$. ~~Point of Inflection~~

$f''(x) = 12\sin(x)\cos^2(x) - 6\sin^3(x) - 3\sin(x)$.

X e) $f(x)$ is concave up on the interval $(0, \pi)$. $= 12\sin(x)(1-\sin^2(x)) - 6\sin^3(x) - 3\sin(x)$

Question 10

Given the graph of $f'(x)$ below, where is $f(x)$ decreasing?

$= -18\sin^3(x) + 9\sin(x)$

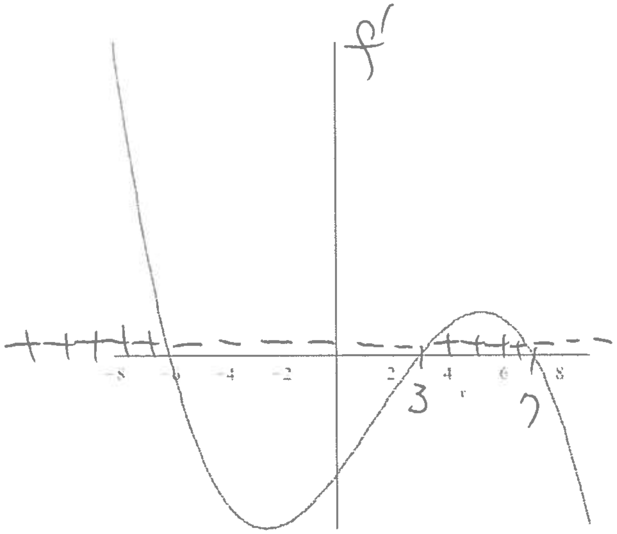
$= 9\sin(x)[2\sin^2(x) - 1]$

point of inflection

$f''(x) = 0$, $\sin(x) = 0 \Rightarrow x = 0$, or $\sin^2(x) = \frac{1}{2} \Rightarrow \sin(x) = \pm\frac{1}{\sqrt{2}}$, $x = \frac{\pi}{4}, \frac{3\pi}{4}$

down | up | down

$0 \;\; - - - \;\; \frac{\pi}{4} \;\; + + + \;\; \frac{3\pi}{4} \;\; - - - \;\; \pi$



Dereasing Intervals : (-6, 3) ∪ (7, ∞).

a) $f(x)$ is decreasing on the interval $(-6, 7)$.

b) $f(x)$ is decreasing on the intervals $(-6, 3)$ and $(7, \infty)$.

c) $f(x)$ is decreasing on the interval $(-6, \infty)$.

d) $f(x)$ is decreasing on the intervals $(-\infty, -6)$ and $(3, 7)$.

e) $f(x)$ is decreasing on the interval $(-\infty, 7)$.