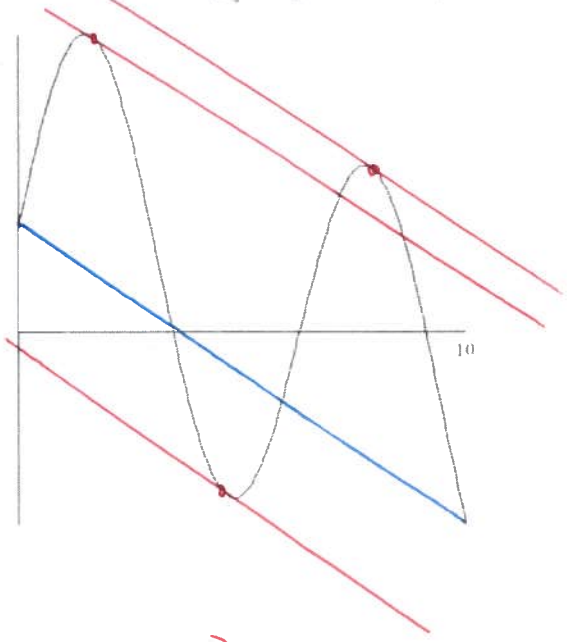
PRINTABLE VERSION

Quiz 11 Sol

Question 1

The function f is graphed below on the interval [0,10].Give the **number of valuesc** between 0 and 10 which satisfy the conclusion of the mean value theorem for f.



3 points

a) 2

b) 4

c) 3

d) 1

e) 5

Question 2

$f'(x)=3x^2-9$

Determine if Rolles Theorem applies to the function $f(x) = x^3 - 9x$ on $[-3, 0]$. If so, find all numbers c on the interval that satisfy the theorem.

a b

$f(a)=f(-3)=0,\ f(b)=f(0)=0$ ✓

a) $c = \sqrt{3}$

$3c^2-9=f'(c)=0 \Rightarrow 3(c+\sqrt{3})(c-\sqrt{3})=0$

b) $c = \sqrt{3}$ and $c = -\sqrt{3}$

$c=\cancel{\sqrt{3}}$ or $-\sqrt{3}$

c) $c = -3$

d) Rolles Theorem does not apply to this function on the given interval.

e) $c = -\sqrt{3}$

Question 3

$f'(x)=\frac{4}{2}\cdot\frac{1}{\sqrt{x}}-1=\frac{2}{\sqrt{x}}-1$

Determine if the function $f(x) = 4\sqrt{x} - x$ satisfies the Mean Value Theorem on [1, 25]. If so, find all numbers c on the interval that satisfy the theorem.

$f(a)=f(1)=3 \quad f(25)=f(b)=4\cdot5-25=-5$

a) $c = -9$

$\frac{2}{\sqrt{c}}-1=f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{-5-3}{25-1}=\frac{-1}{3}.$

b) $c = 9$

$\Rightarrow \frac{2}{\sqrt{c}}=-\frac{1}{3}+1 \Rightarrow \frac{2}{\sqrt{c}}=\frac{2}{3} \Rightarrow c=9$

c) The Mean Value Theorem does not apply to this function on the given interval.

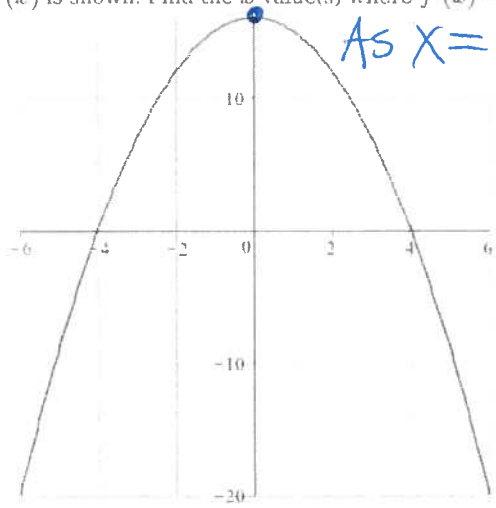
d) $c = \frac{9}{2}$

e) $c = 18$

Question 4

The graph of $f(x)$ is shown. Find the x-value(s) where $f'(x) = 0$.

As $x=0$ $f'(x)=0$.



a) $x = 0$

b) $x = \{-4, 0, 4\}$

c) $x = -4$

d) $x = 4$

e) $x = \{-4, 4\}$

Question 5

Find the intervals on which $f(x) = x^4 + 5x^3$ increases. $\Rightarrow f'(x) > 0$

$f'(x) = 4x^3 + 15x^2 = x^2(4x+15) = 0$

$f'(x)$: $-$ | $+$ | $+$ (at $-\frac{15}{4}$ and 0)

$(-\frac{15}{4}, 0) \cup (0, \infty)$

a) $(-\infty, \infty)$

b) $\left(-\infty, -\frac{15}{4}\right)$

c) $(0, \infty)$

d) $\left(-\frac{15}{4}, 0\right) \cup (0, \infty)$

e) $\left(-\infty, -\frac{15}{4}\right) \cup \left(0, \frac{15}{4}\right)$

Q6. Since $x^2+36 > 0$ $\forall x \in \mathbb{R}$.

the domain $f(x) = \{x \in \mathbb{R}\}$

$f'(x) = \frac{5(x^2+36) - 2x \cdot 5x}{(x^2+36)^2}$

$= \frac{-5x^2 + 5 \cdot 36}{(x^2+36)^2}$

$= \frac{-5(x^2-36)}{(x^2+36)^2} = 0$

$\Leftrightarrow (x^2-36) = 0$

$\Leftrightarrow (x+6)(x-6) = 0$

$\Leftrightarrow x = 6$ or -6

Question 6

Find the intervals on which $f(x) = \frac{5x}{x^2+36}$ decreases.

a) $(-6, 6)$

b) $(-\infty, -6) \cup (0, 6)$

c) $(-\infty, -6) \cup (6, \infty)$

d) $(-\infty, \infty)$

e) $(6, \infty)$

$f'(x)$: $-$ | $+$ | $-$ (at -6 and 6)

decreasing $(-\infty, -6) \cup (6, \infty)$.

Question 7

Find the intervals on which $f(x) = \frac{25x^2+9}{25x^2-9}$ increases.

Q7. As $25x^2-9=0$, $x=\pm\frac{3}{5}$, $f(x)$ DNE.

Domain of f is $\{x\in\mathbb{R} \mid x\neq\pm\frac{3}{5}\}$

$$f'(x)=\frac{50x(25x^2-9)-50x(25x^2+9)}{[25x^2-9]^2}=\frac{-900x}{[25x^2-9]^2}=0$$

$\Leftrightarrow x=0$

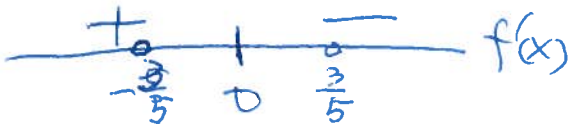
a) $\left(0,\frac{3}{5}\right)\cup\left(\frac{3}{5},\infty\right)$

b) $\left(\left(-\infty,-\frac{3}{5}\right)\cup\left(\frac{3}{5},\infty\right)\right.$

c) $\left(-\infty,-\frac{3}{5}\right)\cup\left(\frac{3}{5},\infty\right)$

d) $\left(-\infty,-\frac{3}{5}\right)\cup\left(-\frac{3}{5},0\right)$

e) $(-\infty,\infty)$



$\Rightarrow$ Increasing interval

$(-\infty,-\frac{3}{5})\cup(-\frac{3}{5},0)$

Question 8

$f'(x)=4x(6+x)^2+2x^2\cdot 2(6+x)$

Find the intervals on which $f(x)=2x^2(6+x)^2$ increases.

$=4x(6+x)[6+x+x]$

$=4x(6+x)(6+2x)=0\Rightarrow x=0,-6,-3$

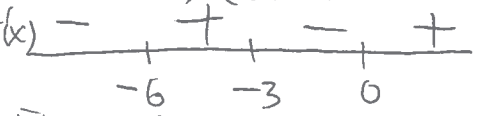
a) $(3,\infty)$

b) $(-6,-3)\cup(0,\infty)$

c) $(-\infty,-6)\cup(3,\infty)$

d) $(-\infty,\infty)$

e) $(-\infty,-6)\cup(-3,0)$



Increasing interval:

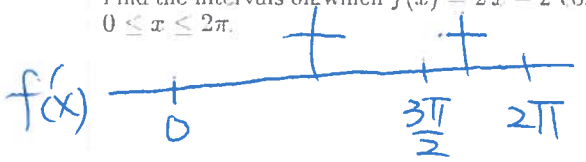
$(-6,-3)\cup(0,+\infty)$

Question 9

$f(x)=2+2\sin(x)=0\Rightarrow\sin(x)=-1.$

$\Rightarrow x=\frac{3\pi}{2}$

Find the intervals on which $f(x)=2x-2\cos(x)$ increases for $0\le x\le 2\pi$.



$\Rightarrow[0,2\pi]$

a) $\left[0,\frac{3\pi}{2}\right]$

b) $[0,2\pi]$

c) $f(x)$ is never increasing on the given interval.

d) $\left[\frac{3\pi}{2},2\pi\right]$

e) $\left[\frac{\pi}{2},2\pi\right]$

$f'(x)=14\sin(x)\cos(x)$

$=7\sin(2x)=0$

$2x=0,\pi,2\pi,$

$x=0,\frac{\pi}{2},\pi.$

Question 10

Find the intervals on which $f(x)=7\cos^2(x)$ decreases for $0\le x\le\pi$.

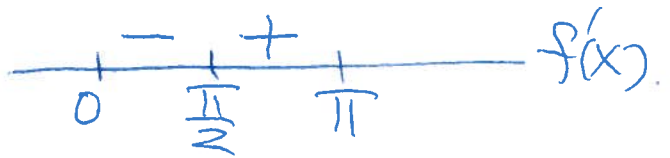
a) $\left[\frac{\pi}{2},\pi\right]$

b) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$

c) $\left[0,\frac{\pi}{2}\right]$

d) $[0,\pi]$

e) $f(x)$ is never decreases on the given interval.



decreasing interval:

$[\frac{\pi}{2},\pi]$