PRINTABLE VERSION

Quiz 10 Sol.

Question 1

Find the rate of change of the volume of a sphere with respect to the radius r . What is the rate when $r = 6$?

$V = \frac{4}{3}\pi r^3$, Find $\left.\frac{dV}{dr}\right|_{r=6}$.

a) $\frac{dV}{dr} = 4r^2\pi$; rate $= 144\pi$

b) $\frac{dV}{dr} = 2r\pi$; rate $= 12\pi$

c) $\frac{dV}{dr} = 4r\pi$; rate $= 24\pi$

d) $\frac{dV}{dr} = 2r^2\pi$; rate $= 72\pi$

e) $\frac{dV}{dr} = 4r^2\pi$; rate $= 72\pi$

do "$\frac{d}{dr}$" on both sides.

$\boxed{\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)}$

$= \frac{4}{3} \cdot 3r^2\pi \cdot \cancel{\frac{dr}{dr}}^{1}$

$= \boxed{4r^2\pi}$, and $\boxed{\left.\frac{dV}{dr}\right|_{r=6} = 4 \cdot 36\pi = 144\pi}$

Question 2

Find the rate of change of the surface area of a sphere with respect to the radius r. What is the rate when $r = 9$?

$V = 4\pi r^2$, Find $\left.\frac{dV}{dr}\right|_{r=9}$.

a) $\frac{dV}{dr} = 4r^2\pi$; rate $= 324\pi$

b) $\frac{dV}{dr} = 8r^2\pi$; rate $= 648\pi$

c) $\frac{dV}{dr} = 8r\pi$; rate $= 72\pi$

d) $\frac{dV}{dr} = 2r\pi$; rate $= 18\pi$

e) $\frac{dV}{dr} = 4r\pi$; rate $= 36\pi$

do "$\frac{d}{dr}$" on both sides,

$\boxed{\frac{dV}{dr} = \frac{d}{dr}(4\pi r^2)} = 4\pi \cdot 2r \cdot \cancel{\frac{dr}{dr}}^{1}$

$= \boxed{8\pi r}$

$\boxed{\left.\frac{dV}{dr}\right|_{r=9} = 8 \cdot 9\pi = 72\pi}$

Question 3

An object moves along a coordinate line, its position at each time $t \geq 0$ is given by $x(t) = (t^2 - 3t)(t^2 + 3t)$. Find the velocity at time $t_0 = 3$.

Velocity = (position)$'$

a) 756

b) 54

c) -54

d) -14

e) 0

$x'(t) = (2t-3)(t^2+3t) + (t^2-3t)(2t+3)$

product rule

$x'(3) = (2\cdot3-3)(3^2+9) + (3^2-3\cdot3)(2\cdot3+3)$

$\boxed{=54}$

Question 4

An object moves along the x-axis, its position at each time $t \geq 0$ is given by $x(t) = -\frac{1}{4}t^4 + 6t^2$. Determine the time interval(s), if any, during which the object slows down. Acceleration is less than 0. but velocity>0

a) $(3, 2)$

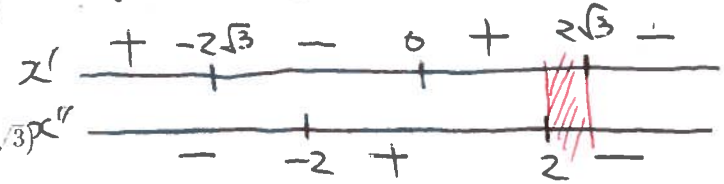
b) $(2, 2\sqrt{3})$

c) $(2, \sqrt{3})$

d) $(2\sqrt{3}, 4\sqrt{3})$

$x'(t) = -t^3 + 12t > 0$, $x''(t) = -3t^2 + 12 < 0$

$-t(t+2\sqrt{3})(t-2\sqrt{3}) > 0$, $-3(t+2)(t-2) < 0$



But $t \geq 0$

$\boxed{\Rightarrow x \in (2, 2\sqrt{3})}$

e) $(2, 4\sqrt{3})$

Question 5

$g = 9.8\ m/s^2 = 32.2\ \text{feet}/s^2$

An object projected vertically upward from ground level returns to earth in 4 seconds. Find the initial velocity in feet per second.

2 seconds from ground to Top.

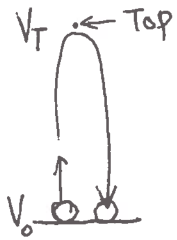
a) 60

b) 64

c) 74

d) 70

e) 80



$V_T = 0$. Find V_0.

$V_0 = V_T + gt$

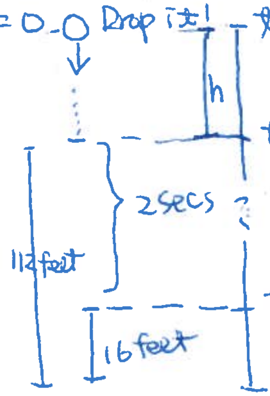
$= 0 + 32.2 \times 2$

$= \boxed{64.4}$

Question 6

A falling stone is at a certain instant 112 feet above the ground and 2 seconds later it is only 16 feet above the ground. From what height was it dropped?

$V_i = 0$. Drop it! $t = 0$



a) 116 feet

b) 118 feet

c) 114 feet

d) 119 feet

e) 117 feet

$? = 112 + h$

$h = V_0 t + \frac{1}{2} g t^2$

$= 0 + \frac{1}{2}(32.2) t_1^2$

$h + (112 - 16) = 0 + \frac{1}{2}(32.2)(t_1 + 2)^2$

$\frac{1}{2}(32.2) t_1^2 + 96 = \frac{1}{2}(32.2)(t_1 + 2)^2$

$\Rightarrow 32.2 t_1 + 64.4 = 96$

$\Rightarrow t_1 = \frac{21.6}{32.2} \Rightarrow h = 7.25$

$? = 112 + 7.25 = \boxed{119}$

Question 7

A particle is moving along the parabola $y^2 = 4(x+5)$. As the particle

passes through the point $(-1, 4)$, the rate of change of its y-coordinate is 3 units per second. How fast, in units per second, is the x-coordinate changing at this instant?

Given $y^2 = 4(x+5)$ and $\left.\frac{dy}{dt}\right|_{(-1,4)} = 3\ \frac{\text{units}}{\text{second}}$.

Find $\frac{dx}{dt}$.

do "$\frac{d}{dt}$" on both sides, $2y \cdot \frac{dy}{dt} = 4 \cdot \frac{dx}{dt} + 0$.

$\Rightarrow \frac{d}{dt}(-1,4)$ $2 \cdot 4 \left.\frac{dy}{dt}\right|_{(-1,4)} = 4 \cdot \left.\frac{dx}{dt}\right|_{(-1,4)}$

$\Rightarrow \frac{2 \cdot 4 \cdot 3}{4} = \boxed{\left.\frac{dx}{dt}\right|_{(-1,4)}}$

$= \boxed{6}$

a) 10

b) 6

c) $\frac{5}{2}$

d) 4

e) $\frac{8}{3}$

Question 8

A heap of rubbish in the shape of a cube is being compacted into a smaller cube. Given that the volume decreases at a rate of 5 cubic meters per minute, find the rate of change of an edge, in meters per minute, of the cube when the volume is exactly 27 cubic meters.

$\frac{dV}{dt} = -5\ \frac{m^3}{min.}$ decrease. let l be the length of edge then $V = l^3$.

Find $\left.\frac{dl}{dt}\right|_{V=27} = \left.\frac{dl}{dt}\right|_{l=3}$ (since $V = l^3$)

do "$\frac{d}{dt}$" on "$V = l^3$" $\Rightarrow \frac{dV}{dt} = 3l^2 \frac{dl}{dt}$

$\Rightarrow -5 = 3 \cdot (3)^2 \left.\frac{dl}{dt}\right|_{l=3} \Rightarrow \boxed{-\frac{5}{27}} = \boxed{\left.\frac{dl}{dt}\right|_{l=3}}$

a) -27

b) $\frac{5}{9}$

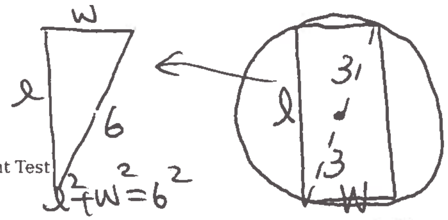
c) $-\frac{27}{5}$

d) $-\frac{5}{27}$

e) 5

Question 9

A rectangle is inscribed in a circle of radius 3 inches. If the length of the



$l^2+w^2=6^2$

let l be the length of rectangle
W be " width "
A be area $\Rightarrow A = lW$.

rectangle is decreasing at the rate of 2 inches per second, how fast is the area changing at the instant when the length is 5 inches?

Find $\left.\frac{dA}{dt}\right|_{l=5}$. (Given $\frac{dl}{dt}=-2$, $A=lW$, $l^2+W^2=6^2$).

$l^2+W^2=36 \Rightarrow W=\sqrt{36-l^2} \Rightarrow A = l\cdot\sqrt{36-l^2}$

do "$\frac{d}{dt}$": $\frac{dA}{dt} = \frac{dl}{dt}\cdot\sqrt{36-l^2} + l\cdot\frac{1}{2}(36-l^2)^{-\frac{1}{2}}\cdot(-2l)\cdot\frac{dl}{dt}$

$\left.\frac{dA}{dt}\right|_{l=5} = -2\cdot\sqrt{36-25} + 5\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{36-25}}\cdot(-2\cdot5)\cdot(-2)$

$= -2\cdot\sqrt{11} + \frac{50}{\sqrt{11}} = \frac{-22+50}{\sqrt{11}} = \boxed{\frac{28}{\sqrt{11}}}$

a) $-10\sqrt{11}\ in^2/sec$

b) $\frac{5\sqrt{11}}{3}\ in^2/sec$

c) $5\sqrt{11}\ in^2/sec$

d) $\frac{-28\sqrt{11}}{11}\ in^2/sec$

e) $\frac{28\sqrt{11}}{11}\ in^2/sec$

Question 10

A spherical snowball is melting in such a manner that its radius is changing at a constant rate, decreasing from 28 cm to 18 cm in 30 minutes. At what rate, in cm^3 per minute, is the volume of the snowball changing at the instant the radius is 4 cm?

$\frac{dr}{dt} = \frac{18-28}{30} = -\frac{1}{3}\left(\frac{cm}{min}\right)$.

$V=\frac{4}{3}\pi r^3$, Find $\left.\frac{dV}{dt}\right|_{r=4}$.

do "$\frac{d}{dt}$": $\frac{dV}{dt} = \frac{4}{3}\pi\cdot3r^2\frac{dr}{dt}$.

$\left.\frac{dV}{dt}\right|_{r=4} = \frac{4}{3}\cdot\pi\cdot3\cdot(4)^2\cdot\left(-\frac{1}{3}\right) = -\frac{128\pi}{3}$

a) $-\frac{128\pi}{3}$

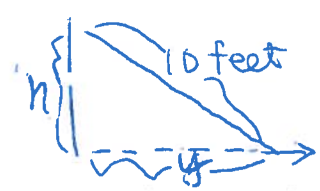
b) $-\frac{64\pi}{3}$

c) $\frac{32\pi}{3}$

d) 160π

e) $-\frac{34\pi}{3}$

Question 11



h: from top of ladder to ground
y: from bottom of the ladder to the wall

A 10-foot ladder is leaning against a vertical wall. If the bottom of the ladder is being pulled away from the wall at the rate of 9 feet per second, at what rate is the area of the triangle formed by the wall, the ground, and the ladder changing, in square feet per second, at the instant the bottom of the ladder is 6 feet from the wall?

$h^2+y^2=100$, $\frac{dy}{dt} = 9\frac{feet}{sec.}$, $A=\frac{1}{2}hy$

$\Rightarrow h=\sqrt{100-y^2}$

Find $\left.\frac{dA}{dt}\right|_{y=6}$

$A=\frac{1}{2}y\sqrt{100-y^2}$

"$\frac{d}{dt}$": $\frac{dA}{dt} = \frac{1}{2}\frac{dy}{dt}\sqrt{100-y^2} + \frac{1}{2}y\cdot\frac{1}{2}(100-y^2)^{-\frac{1}{2}}(-2y)\frac{dy}{dt}$

$\left.\frac{dA}{dt}\right|_{y=6} = \frac{1}{2}\cdot9\cdot\sqrt{100-36} + \frac{1}{2}\cdot6\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{100-36}}\cdot(-2\cdot6)\cdot9$

$= \frac{1}{2}\cdot72 + \frac{-2\cdot6\cdot6\cdot9}{4\cdot8}$

$= 36 - \frac{81}{4} = \frac{63}{4}$

a) $\frac{63}{8}$

b) $\frac{63}{4}$

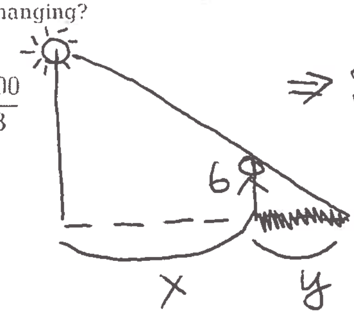
c) $-\frac{63}{4}$

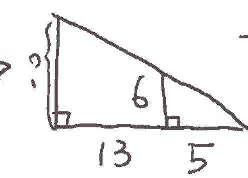
d) $-\frac{63}{2}$

e) $\frac{63}{2}$

Question 12

A man standing 13 feet from the base of a lamppost casts a shadow 5 feet long. If the man is 6 feet tall and walks away from the lamppost at a speed of 450 feet per minute, at what rate, in feet per minute, will the length of his shadow be changing?





$\frac{6}{5} = \frac{?}{13+5}$

$\Rightarrow ? = \frac{6}{5}\cdot18$

a) $-\frac{4500}{13}$

b) $\frac{1125}{13}$

c) $\frac{4500}{13}$

d) $\frac{2250}{13}$

Given $\frac{dx}{dt} = 450\frac{feet}{min}$ Find $\left.\frac{dy}{dt}\right|_{x=13, y=5}$.

$\frac{6}{y} = \frac{\frac{6}{5}\cdot18}{x+y} \Rightarrow 6x+6y = \frac{108}{5}y$

$\Rightarrow 6x = \frac{78}{5}y \Rightarrow 6\cdot\frac{dx}{dt} = \frac{78}{5}\frac{dy}{dt}$

$\Rightarrow 6\cdot450\cdot\frac{5}{78} = \left.\frac{dy}{dt}\right|_{x=13} \Rightarrow \left.\frac{dy}{dt}\right|_{x=13} = \frac{2250}{13}$

e) $-\frac{2250}{13}$