PRINTABLE VERSION

Practice Test 4

Question 1

Differentiate $y = 2\,e^{3\,x}\,\arcsin(x)$. product

$y' = 6e^{3x} \arcsin(x) + 2e^{3x} \cdot \frac{1}{\sqrt{1-x^2}}$

a) $2e^{3x}\arcsin(x) + \dfrac{2e^{3x}}{\sqrt{1-x^2}}$

b) $6e^{3x}\arcsin(x) + \dfrac{2e^{3x}}{\sqrt{1+x^2}}$

c) $6e^{3x}\arcsin(x) + \dfrac{2e^{3x}}{\sqrt{1-x^2}}$

d) $\dfrac{6e^{3x}}{\sqrt{1+x^2}}$

e) $\dfrac{6e^{3x}}{\sqrt{1-x^2}}$

Question 2

Differentiate the given function $y = \cosh\left(\ln\left(6x^4\right)\right)$.

$y' = \sinh(\ln(6x^4)) \cdot \frac{(6x^4)'}{6x^4}$

$= \sinh(\ln(6x^4)) \cdot \frac{24x^3}{6x^4}$

$= \frac{4}{x} \cdot \frac{e^{\ln(6x^4)} - e^{-\ln(6x^4)}}{2}$

$\boxed{\sinh(x) = \frac{e^x - e^{-x}}{2}}$

$= \frac{\cancel{4}^{\,2}}{x} \cdot \frac{6x^4 - \frac{1}{6x^4}}{\cancel{2}} = 2\left(6x^3 - \frac{1}{6x^5}\right) = 12x^3 - \frac{1}{3x^5}$

a) $12x^3 - \dfrac{2}{x^4}$

b) $3x^3 + \dfrac{1}{3\,x^5}$

c) $12x^3 - \dfrac{1}{3x^5}$

d) $3x^3 - \dfrac{4}{x^5}$

e) $4x^3 + \dfrac{1}{3x^4}$

Q3. $y = A\cosh(Cx) + B\sinh(Cx)$

$y' = AC\sinh(Cx) + BC\cosh(Cx)$

$y'' = AC^2\cosh(Cx) + BC^2\sinh(Cx)$

① $y'' - 25y = 0$ implies $AC^2\cosh(Cx) + BC^2\sinh(Cx) - 25(A\cosh(Cx) + B\sinh(Cx)) = 0$

$\Rightarrow (AC^2 - 25A)\cosh(Cx) + (BC^2 - 25B)\sinh(Cx) = 0$

$\Rightarrow \begin{cases} AC^2 - 25A = 0 \\ BC^2 - 25B = 0 \end{cases}$ — (I)

Question 3

Determine A, B, and C so that $y = A\cosh(Cx) + B\sinh(Cx)$ satisfies the conditions $y'' - 25y = 0$, $y(0) = 1$, $y'(0) = 2$ Take $C > 0$.

② $y(0) = 1$ implies $A\cosh(0) + B\sinh(0) = 1$

$\Rightarrow A = 1$ — (II)

③ $y'(0) = 2$ implies $AC\sinh(0) + BC\cosh(0) = 2$

$\Rightarrow BC = 2$

By (I) (II) $C^2 - 25 = 0$, $C = 5$ or $\cancel{-5}$

By (III) $B = \frac{2}{5}$. ✓

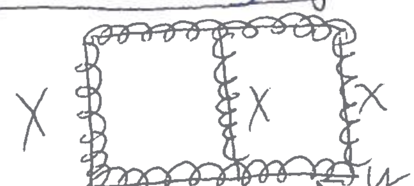
a) $[A = 5/2, B = 2, C = 5]$

b) $[A = 4, B = 2/5, C = 1]$

c) $[A = 3, B = 1/2, C = 5]$

d) $[A = 1, B = 2/5, C = 5]$

e) $[A = 5, B = 5/2, C = 0]$



Question 4

A rectangular playground is to be fenced off and divided into two parts by a fence parallel to one side of the playground. 1080 feet of fencing is used. Find the dimensions of the playground that will enclose the greatest total area.

① max. function xy.

② The relation $3x + 2y = 1080$

③ The restriction $\Rightarrow y = \frac{1080 - 3x}{2}$, $x > 0$, $y > 0$

a) 290 by 190 feet with the divider 190 feet long

b) 270 by 270 feet with the divider 270 feet long

④ Let $f(x) = xy = x\left(\frac{1080 - 3x}{2}\right) = 540x - \frac{3}{2}x^2$.

c) 265 by 185 feet with the divider 266 feet long

$f'(x) = 540 - 3x \Rightarrow f'(x) = 0$ implies $x = \frac{540}{3} = 180$.

0 180 $\Rightarrow$ Max, $x = 180$ by $y = 270$ with 180 divider.

d) 280 by 190 feet with the divider 280 feet long

e) 270 by 180 feet with the divider 180 feet long

Question 5

Find A and B given that the function $y = \dfrac{A}{\sqrt{x}} + B\sqrt{x}$ has a minimum value of 32 at x = 16.

$\Rightarrow$ ① as $x=16$, $y=32$

$\Rightarrow 32 = \dfrac{A}{\sqrt{16}} + B\sqrt{16}$

$= \dfrac{A}{4} + 4B$

② At $x=16$, there is a min.

$y' = -\dfrac{A}{2}\cdot x^{-\frac{3}{2}} + \dfrac{B}{2}x^{-\frac{1}{2}}$

$= \dfrac{-A+Bx}{2x^{\frac{3}{2}}}$

$\Rightarrow -A + B\cdot 16 = 0$, $A = 16B$

By ①② $32 = \dfrac{16B}{4} + 4B = 8B \Rightarrow B = 4$.

$A = 64$.

Check min. number line

$y' = \dfrac{-64+4x}{2x^{\frac{3}{2}}}$

---- 16 ++++ ✓ check!

a) A = 128 and B = 8

b) A = 128 and B = 4

c) A = 64 and B = 12

d) A = 64 and B = 4

e) A = 64 and B = 8

Question 6

Use differentials to estimate the value $(80.8)^{1/4}$.

① find function $f(x) = x^{\frac{1}{4}}$. $\Rightarrow f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$

② Given $a+h = 80.8$, pick up "a" $= 81$ (since $81^{\frac{1}{4}} = 3$)

③ $h = -0.2$

④ $(80.8)^{\frac{1}{4}} = f(a+h) \approx f(a) + f'(a)\cdot h$

$= 81^{\frac{1}{4}} + \frac{1}{4}\cdot\frac{1}{81^{\frac{3}{4}}}\cdot\left(-\frac{2}{10}\right)$

$= 3 + \frac{1}{4}\cdot\frac{1}{27}\cdot\left(-\frac{1}{5}\right)$

$= 3 - \frac{1}{540} = \frac{1619}{540}$

a) $\dfrac{1619}{540}$

b) $\dfrac{1621}{540}$

c) $\dfrac{1349}{540}$

d) $\dfrac{1889}{540}$

Change 58° to radians: $58^\circ = 58 \cdot \frac{\pi}{180} = a + h$.

e) $\frac{14}{5}$

① Find the function $f(x) = \cos(x)$

$\Rightarrow f'(x) = -\sin(x)$.

Question 7

Use differentials to estimate the value $\cos(58^o)$.

② Pick up $a = \frac{\pi}{3}$ (since $\cos(\frac{\pi}{3}) = \frac{1}{2}$).

a) $\frac{1}{2} + \frac{\sqrt{3}}{180}\pi$

③ $h = \frac{58\pi}{180} - \frac{\pi}{3} = \frac{-2\pi}{180} = \frac{-\pi}{90}$

b) $\frac{1}{2} + \frac{\sqrt{3}}{90}\pi$

④ $\cos(58^\circ) = f(a+h) \approx f(a) + f'(a) \cdot h$

c) $\frac{\sqrt{3}}{2} - \frac{1}{180}\pi$

$= \cos(\frac{\pi}{3}) - \sin(\frac{\pi}{3}) \cdot (-\frac{\pi}{90})$

d) $\frac{1}{2} - \frac{\sqrt{3}}{180}\pi$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{-\pi}{90}$

e) $\frac{\sqrt{3}}{2} - \frac{1}{90}\pi$

$= \frac{1}{2} + \frac{\sqrt{3}}{180}\pi$.

Using log differentiation. let $y = (8x+3)^{3x}$.

Question 8

Find the derivative of $(8\,x+3)^{3\,x}$.

① $\ln y = 3x \cdot \ln(8x+3)$.

② do derivative on both sides

$\frac{y'}{y} = 3 \cdot \ln(8x+3) + 3x \cdot \frac{8}{8x+3}$

a) $\left(3\ln(8x+3) + \frac{24\,x}{8\,x+3}\right)$

b) $24x(8x+3)^{3x-1}$

③ $y' = \left[3\ln(8x+3) + \frac{24x}{8x+3}\right](8x+3)^{3x}$

c) $(8\,x+3)^{3\,x}\left(3\ln(8x+3) + \frac{24\,x}{8\,x+3}\right)$

d) $(8\,x+3)^{3\,x}\left(3\ln(8x+3) - \frac{3}{8\,x+3}\right)$

e) $3x(8x+3)^{3x-1}$

Question 9

Calculate the limit: $\lim_{x \to 0} \frac{e^{x} + e^{-x} - 2}{1 - \cos(5x)}$.

$\left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x \to 0} \frac{e^{x} - e^{-x}}{5\sin(5x)}$

$\left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x \to 0} \frac{e^{x} + e^{-x}}{25\cos(5x)}$

$= \frac{e^{0} + e^{0}}{25\cos(0)} = \frac{2}{25}.$

a) 1

b) $\frac{2}{25}$

c) 0

d) $\frac{4}{25}$

e) $\frac{25}{2}$

Question 10

$\lim_{x \to \infty} e^{\ln (x^{9}+1)^{\frac{1}{\ln(x)}}} = \lim_{x \to \infty} e^{\frac{1}{\ln(x)} \ln(x^{9}+1)}$

$\overset{\frown}{=} e^{\lim_{x\to\infty} \frac{\ln(x^{9}+1)}{\ln(x)}} = e^{9}$

$\|$

Calculate the limit: $\lim_{x \to \infty} \left(x^{9} + 1\right)^{\frac{1}{\ln(x)}}$.

exp function is conti.

and $\lim_{x \to \infty} \frac{\ln(x^{9}+1)}{\ln(x)} \left(\frac{\infty}{\infty}\right) \overset{L'}{=} \lim_{x \to \infty} \frac{\frac{9x^{8}}{x^{9}+1}}{\frac{1}{x}}$

$= \lim_{x \to \infty} \frac{9x^{9}}{x^{9}+1} = 9$

a) $-e^{9}$

b) e^{10}

c) $-e^{10}$

d) e^{9}

e) 0

Question 11

U_f

Compute the upper Riemann sum for the given function $f(x) = \sin(x)$ over the interval $x \in [0, \pi]$ with respect to the partition $P = \left[0, \frac{\pi}{3}, \frac{5\pi}{6}, \pi\right]$.

subinterval	length	max value
$[0, \frac{\pi}{3}]$	$\frac{\pi}{3}$	$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$
$[\frac{\pi}{3}, \frac{5\pi}{6}]$	$\frac{\pi}{2}$	$\sin(\frac{\pi}{2}) = 1$
$[\frac{5\pi}{6}, \pi]$	$\frac{\pi}{6}$	$\sin(\frac{5\pi}{6}) = \frac{1}{2}$

a) $\frac{5}{12}\pi + \frac{\sqrt{3}}{12}\pi$

b) $\frac{17}{36}\pi + \frac{\sqrt{3}}{9}\pi$

c) $\frac{1}{4}\pi$

d) $\frac{13}{36}\pi + \frac{\sqrt{3}}{18}\pi$

e) $\frac{7}{12}\pi + \frac{\sqrt{3}}{6}\pi$

$$U_f = \frac{\pi}{3}\cdot\frac{\sqrt{3}}{2} + \frac{\pi}{2}\cdot 1 + \frac{\pi}{6}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{3}}{6}\pi + \frac{7}{12}\pi$$

Question 12

Given that $\int_0^1 f(x)\,dx = 4$, $\int_0^4 f(x)\,dx = 6$ and $\int_4^5 f(x)\,dx = 3$ find $\int_5^1 f(x)\,dx$.

$$\Rightarrow \int_5^1 f(x)dx = -\int_1^5 f(x)dx$$
$$= -\left[\int_4^5 f(x)\,dx + \int_0^4 f(x)dx - \int_0^1 f(x)dx\right]$$
$$= -[3 + 6 - 4] = -5$$

a) -3

b) -1

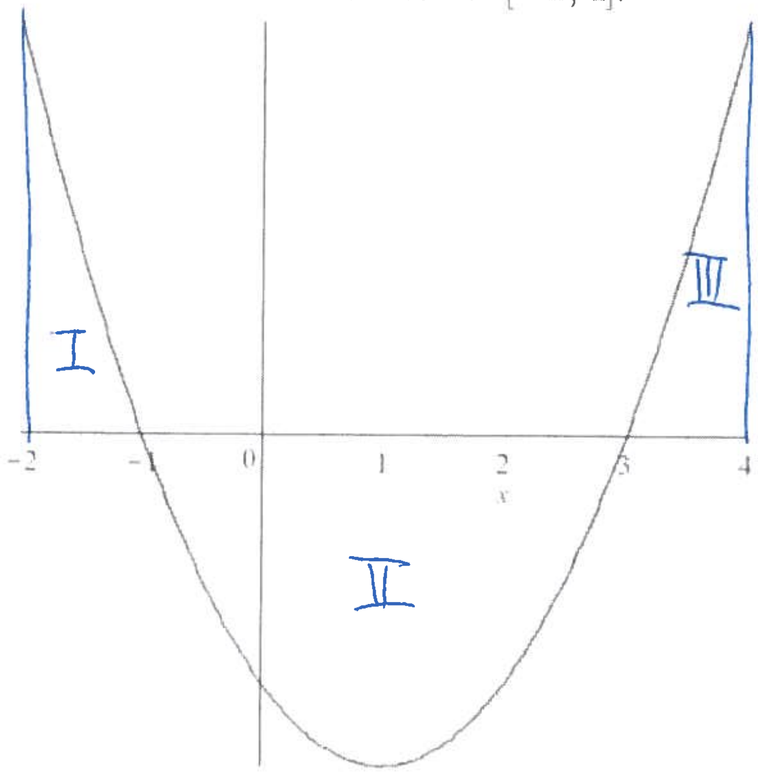
c) 5

d) 3

e) -5

Question 13

The graph of f is shown below on the interval $[-2, 4]$.



The area bounded between the graph of f and the x-axis on $[-2, -1]$ is $\frac{7}{3}$, $=$ Area(I)
the area bounded between the graph of f and the x-axis on $[-1, 3]$ is $\frac{32}{3}$, $=$ Area(II)
and the area bounded between the graph of f and the x-axis on $[3, 4]$ is $\frac{7}{3}$. $=$ Area(III)

Determine $\int_{-2}^{-1} f(x)\,dx$ $=$ Area(I) $= \frac{7}{3}$

a) $\frac{7}{3}$

b) 0

c) $\frac{46}{3}$

d) $-\frac{7}{3}$

e) 13

By F.T.C. we do derivative on both sides, we get

$6x^5+4x^3+7=f(x)$.

Question 14

Find a formula for $f(x)$ given that f is continuous and

$x^6+x^4+7x=\int_0^x f(t)\,dt.$

a) $f(x)=x^6+x^4+8x$

b) $f(x)=1/7\,x^7+1/5\,x^5+7/2\,x^2+7$

c) $f(x)=x^6+x^4+7x$

d) $f(x)=1/7\,x^7+1/5\,x^5+7/2\,x^2$

e) $f(x)=6x^5+4x^3+7$

$$|x-3|=\begin{cases} x-3, & x-3>0; \\ -(x-3), & x-3<0, \end{cases}$$

$$=\begin{cases} x-3, & x>3 \\ 3-x, & x<3. \end{cases}$$

Question 15

Evaluate the definite integral: $\int_1^4 |x-3|\,dx$

$$=\int_1^3 (3-x)dx+\int_3^4 (x-3)dx$$

$$=\left[3x-\frac{x^2}{2}\right]_1^3+\left[\frac{x^2}{2}-3x\right]_3^4$$

$$=3(3-1)-\frac{1}{2}(3^2-1^2)+\frac{1}{2}(4^2-3^2)-3(4-3)$$

$$=6-4+\frac{7}{2}-3=\frac{5}{2}$$

a) -1

b) $\frac{5}{2}$

c) $\frac{33}{2}$

d) $-\frac{111}{2}$

e) $-\frac{3}{2}$

Question 16

Find $\int_{-3}^{4} f(x)\,dx$ given that $f(x) = \begin{cases} x+2 & -3 \le x \le 0 \\ 2 & 0 < x \le 1 \\ 4-2x & 1 < x \le 4 \end{cases}$

$\|$

$\int_{-3}^{0}(x+2)dx + \int_0^1 2\,dx + \int_1^4 (4-2x)dx$

$= \left[\frac{x^2}{2}+2x\right]_{-3}^{0} + \left[2x\right]_0^1 + \left[4x-x^2\right]_1^4$

$= \frac{1}{2}\left((0)^2-(-3)^2\right)+2(0-(-3)) + 2(1-0) + 4(4-1) - \left((4)^2-1^2\right)$

$= -\frac{9}{2}+6+2+12-15 = \frac{1}{2}$

a) $\frac{1}{2}$

b) -3

c) $\frac{35}{2}$

d) -21

e) 21

Question 17

Calculate the indefinite integral: $\int \frac{2x^3-5}{x^2}\,dx.$ split it $\overset{\Downarrow}{=} \int \frac{2x^3}{x^2} - \frac{5}{x^2}dx$

$= \int \left(2x - \frac{5}{x^2}\right)dx$

$= x^2 + \frac{5}{x} + C$

a) $x^2 + \frac{5}{x} + C$

b) $x^2 - 5x + C$

c) $6 - \frac{4x^3-10}{x^3} + C$

d) $\frac{2}{3}x^3 - 5x + C$

e) $2x + \frac{5}{x} + C$

Question 18

Calculate the indefinite integral: $\int \left(5x^3 + 2\sqrt{x} + \frac{1}{x^3}\right) dx.$

$= \frac{5}{4}X^4 + 2\frac{X^{\frac{3}{2}}}{\frac{3}{2}} + \frac{X^{-2}}{-2} + C$

$= \frac{5}{4}X^4 + \frac{4}{3}X^{\frac{3}{2}} - \frac{1}{2X^2} + C.$

a) $15x^2 + \frac{1}{\sqrt{x}} - \frac{3}{x^4} + C$

b) $\frac{5}{4}x^4 + \frac{4}{3}x^{3/2} - \frac{1}{x} + C$

c) $\frac{5}{3}x^3 - \frac{4}{3}x^{3/2} - \frac{1}{2\,x^2} + C$

d) $\frac{5}{4}x^4 + \frac{4}{3}x^{3/2} - \frac{1}{2\,x^2} + C$

e) $\frac{5}{4}x^4 - \frac{4}{3}x^{3/2} - \frac{1}{2\,x^2} + C$

Question 19

Find f givent that $f'(x) = 4x - 6$ and $f(1) = 1$.

$f(x) = \int (4X - 6)dX = 2X^2 - 6X + C.$

$1 = f(1) = 2(1)^2 - 6\cdot 1 + C = -4 + C.$

$C = 5.$

$\Rightarrow f(x) = 2X^2 - 6X + 5.$

a) $f(x) = 4x - 1$

b) $f(x) = 4x + 2$

c) $f(x) = 2x^2 - 6x + 5$

d) $f(x) = 2x^2 - 6x + 8$

e) $f(x) = 2x^2 - 6x + 2$

Question 20

Calculate: $\int \sec(2\,x + 4)\tan(2\,x + 4)\,dx$

Let $u = 2x+4$, $du = 2dx$.

$\Rightarrow \frac{du}{2} = dx$

$= \int \sec(u) \cdot \tan(u) \frac{du}{2}$

$= \frac{1}{2}\int \sec(u)\tan(u)du$

$= \frac{1}{2}\sec(u) + C$

$= \frac{1}{2}\sec(2x+4) + C$

a) $\frac{1}{2}\sec(2x+4)\tan(2x+4) + C$

b) $\frac{1}{2}\sec(2x+4) + C$

c) $\frac{1}{2}\tan(2x+4) + C$

d) $2\tan(2x+4) + C$

e) $2\sec(2x+4) + C$