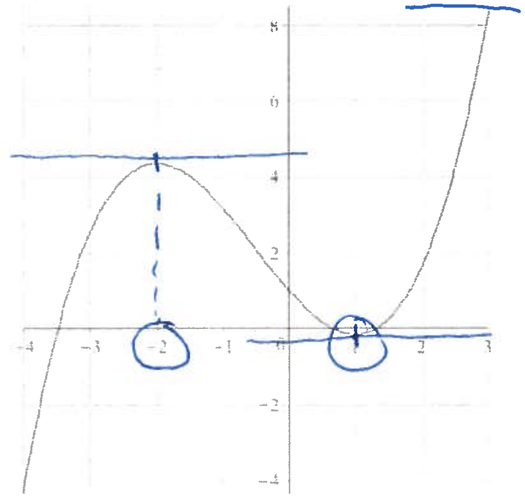
PRINTABLE VERSION

Practice Test 3

Sol

Question 1

The graph of $f(x)$ is shown. Find the x-value(s) where $f'(x) = 0$. ⇒ The places f has horizontal tangent line.



$X = -2$ or $X = 1$.

a) $x = -2$

b) $x = \{-2, 1\}$

c) $x = 0$

d) $x = \{-2, 0, 1\}$

e) $x = \{-2, 2\}$

Question 2

Find the intervals on which $f(x) = \dfrac{4x}{x^2+81}$ decreases.

$D(f) = \mathbb{R}$ (since $x^2+81>0$)

$f'(x) = \dfrac{4(x^2+81) - 2x \cdot 4x}{(x^2+81)^2} = \dfrac{-4x^2+4\cdot 81}{(x^2+81)^2} = \dfrac{-4(x^2-81)}{(x^2+81)^2} = \dfrac{-4(x+9)(x-9)}{(x^2+81)^2}$

a) $(-\infty, -9) \cup (9, \infty)$

b) $(-\infty, \infty)$

c) $(-\infty, -9) \cup (0, 9)$

d) $(9, \infty)$

e) $(-9, 9)$

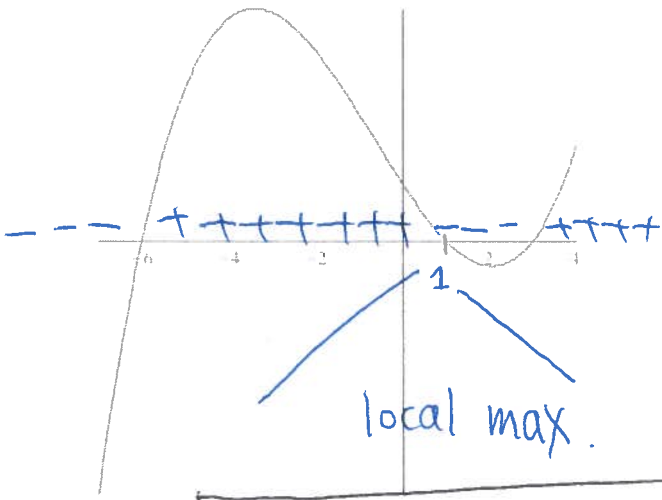
$- - - - \; -9 \; + + + + \; 9 \; - - -$

$(-\infty, -9) \cup (9, \infty)$

Question 3

Suppose that $c = 1$ is a critical number for a function f. Determine if $f(c)$ is a local maximum, local minimum or neither if the graph of $f'(x)$ is shown below.

$f'(1) = 0$



local max.

Q4. $f'(x)=8x^3-8x$

$D(f)=\mathbb{R}$ $=8x(x^2-1)=8x(x+1)(x-1)$

a) Local Minimum

b) Local Maximum

c) Neither

Question 4

Find the critical numbers of $f(x) = 2x^4 - 4x^2 + 1$ and classify all local extreme values.

local min local max local min

a) Critical nos. ± 1; local min $f(-1) = -1$; local max $f(1) = -1$.

$f(-1)=-1$ $f(0)=1$ $f(1)=-1$

b) Critical no. 0; local max $f(0) = 1$.

c) No critical numbers, no extreme values.

d) Critical nos. 0 and ± 1; local min $f(-1) = -1$ and $f(1) = -1$; local max $f(0) = 1$.

e) Critical nos. ± 1; local min $f(1) = -1$, local max $f(-1) = -1$.

Question 5

$f'(x)=8x+2=0$, $x=-\frac{1}{4}$.

Find the critical numbers of $f(x) = 4x^2 + 2x + 1$ and classify all extreme values given $-1 \le x \le 0$.

$f(0)=1$.

a) Critical no. 0; local min $f(0) = 1$.

b) Critical no. $-\frac{1}{4}$; local and absolute min $f\left(-\frac{1}{4}\right) = \frac{3}{4}$; absolute max $f(-1) = 3$.

$f(-1)=3$ local min $f(-\frac{1}{4})=\frac{3}{4}$

c) No critical numbers, no extreme values.

abs. max

d) Critical nos. 0, $-\frac{1}{4}$; local and absolute min $f\left(-\frac{1}{4}\right) = \frac{3}{4}$; absolute max $f(0) = 1$.

abs. min

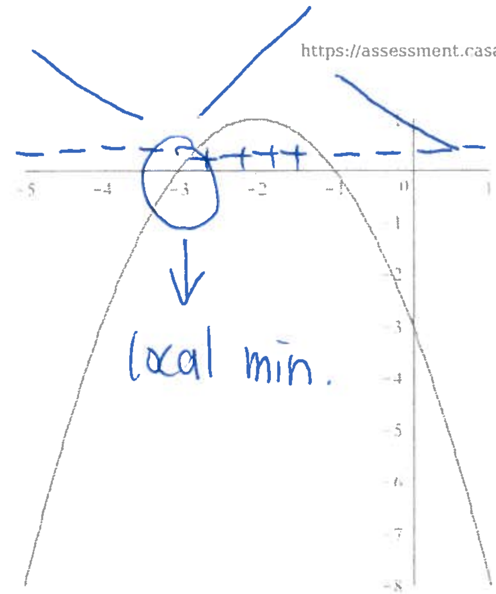
e) Critical no. $-\frac{1}{4}$; local max $f\left(-\frac{1}{4}\right) = \frac{3}{4}$; no absolute extreme.

Question 6

Read Carefully! The graph of f' (the derivative of f) is shown below. Classify the smallest critical number for f. implies $f'(x)=0$

$\Rightarrow x=-3$. or -1

Smallest one $\Rightarrow x=-3$.



a) local maximum

b) neither

c) local minimum

Q7 $D(f) = \{9x^2 - 16 \neq 0\} = \{x \neq \pm \frac{4}{3}\}$

$$f'(x) = \frac{2\cdot(9x^2-16) - 2x\cdot(18x)}{(9x^2-16)^2}$$

$$= \frac{-18x^2 - 32}{(9x^2-16)^2}$$

Question 7

Describe the concavity of the graph of $f(x) = \dfrac{2x}{9x^2 - 16}$ and find the points of inflection (if any).

a) concave down on $\left(-\infty, \frac{4}{3}\right)$; concave up on $\left(\frac{4}{3}, \infty\right)$; pt of inflection $\left(\frac{4}{3}, 0\right)$.

$$f''(x) = \frac{-36x(9x^2-16)^2 - 2\cdot 18x(9x^2-16)\cdot(-18x^2-32)}{(9x^2-16)^4}$$

$$= \frac{-36x(9x^2-16)\left[9x^2-16-18x^2-32\right]}{(9x^2-16)^4} = \frac{-36x\left[-9x^2-48\right]}{(9x^2-16)^3} = \frac{36x(9x^2+48)}{(9x^2-16)^3}$$

$\Rightarrow x = 0$ has an point of inflection $\Rightarrow (0,0)$

down up down up

$- - - - \; -\frac{4}{3} \; + + + + \; 0 \; - - - - \; \frac{4}{3} \; + + + +$

b) concave down on $(-\infty, \infty)$; no points of inflection

c) concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; pt of inflection $(0, 0)$.

d) concave down on $\left(-\infty, -\frac{4}{3}\right)$ and $\left(0, \frac{4}{3}\right)$; concave up on $\left(-\frac{4}{3}, 0\right)$ and $\left(\frac{4}{3}, \infty\right)$; pt of inflection $(0, 0)$.

e) concave up on $\left(-\frac{4}{3}, \frac{4}{3}\right)$; concave down on $\left(-\infty, -\frac{4}{3}\right)$ and $\left(\frac{4}{3}, \infty\right)$; pts of inflection $\left(-\frac{4}{3}, 0\right)$ and $\left(\frac{4}{3}, 0\right)$.

Question 8

Find c so that the graph of $f(x) = cx^2 - 4x^{-2}$ has a point of inflection at $(4, f(4))$. $\Rightarrow f''(4) = 0$.

a) $c = \frac{3}{64}$

b) $c = \frac{3}{32}$

c) $c = -\frac{3}{64}$

d) $c = 0$

e) $c = -\frac{3}{32}$

$f'(x) = 2cx + 8x^{-3}$.

$f''(x) = 2c - 24x^{-4}$.

$f''(4) = 0 \Rightarrow 2c - \frac{24}{(4)^4} = 0$

$c = \frac{3}{64}$

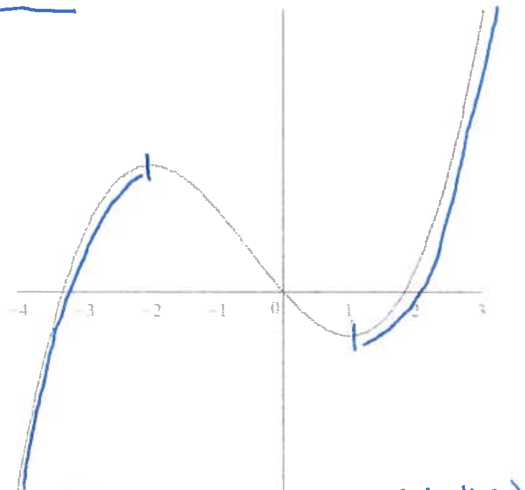
Question 9

The graph of $f'(x)$ is shown below. Give the interval(s) where the graph of

f' increasing $\Rightarrow f'' > 0 \Rightarrow$ concave up

f' decreasing $\Rightarrow f'' < 0 \Rightarrow$ concave down.

$f(x)$ is concave up.



$\Rightarrow (-\infty, -2) \cup (1, \infty)$

a) $(-2, 1)$

b) $(-\infty, 0)$ and $(1, \infty)$

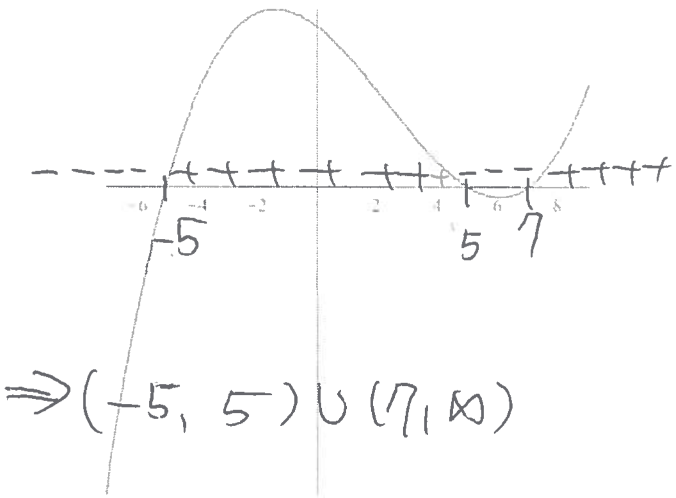
c) $(0, \infty)$

d) $(-\infty, 0)$

e) $(-\infty, -2)$ and $(1, \infty)$

Question 10

Given the graph of $f'(x)$ below, where is $f(x)$ increasing? $\Rightarrow f'(x) > 0$



$\Rightarrow (-5, 5) \cup (7, \infty)$

a) $f(x)$ is increasing on the interval $(-5, \infty)$.

b) $f(x)$ is increasing on the intervals $(-\infty, -5)$ and $(5, 7)$.

c) $f(x)$ is increasing on the interval $(-\infty, 7)$.

d) $f(x)$ is increasing on the intervals $(-5, 5)$ and $(7, \infty)$.

e) $f(x)$ is increasing on the interval $(-5, 7)$.

Question 11

Find the vertical and horizontal asymptotes of $f(x) = \dfrac{2x}{2x - 3}$.

V.A. $f(x) \to \pm\infty$ as $x \to \frac{3}{2}^{\pm}$ $(2x - 3 = 0)$

H.A. $f(x) \to 1$ as $x \to \infty$

$\Rightarrow$ V.A. $\Rightarrow x = \frac{3}{2}$

H.A $\Rightarrow y = 1$.

a) vertical asymptote: $x = \frac{3}{2}$; no horizontal asymptote.

b) vertical asymptote: $x = 1$; horizontal asymptote: $y = \frac{3}{2}$.

c) vertical asymptote: $x = \frac{3}{2}$; horizontal asymptote: $y = 0$.

d) vertical asymptote: $x = \frac{3}{2}$; horizontal asymptote: $y = 1$.

e) no vertical asymptote; horizontal asymptote: $y = 1$.

Question 12

Determine whether or not the graph of $f(x) = 2(x-4)^{4/5}$ has a vertical tangent or vertical cusp at $x = 4$.

$f'(x) = 2 \cdot \frac{4}{5}(x-4)^{-\frac{1}{5}} = \frac{8}{5} \frac{1}{(x-4)^{\frac{1}{5}}}$

a) vertical tangent

b) vertical cusp

c) both

d) neither

$\lim_{x \to 4^-} f'(x) = -\infty$

$\lim_{x \to 4^+} f'(x) = \infty$

Vertical $\Rightarrow$ cusp

Question 13

Which of the following is true about the graph of $f(x) = 27x^2 - \frac{54}{x} - 4$?

Q13, $D(f) = \{x \neq 0\}$, $f'(x) = 54x - \frac{54}{x^2}$

critical number: $x = 1$

f' – – – – 0 – – – 1 + + + +

$(-\sqrt[3]{2}, f(-\sqrt[3]{2}))$

a) $f(x)$ has a point of inflection at the point $(0, -4)$.

b) $f(x)$ is concave down on the interval $(0, \infty)$.

$\Rightarrow (-\sqrt[3]{2}, 0)$

$f''(x) = 54 + \frac{108}{x^3} = \frac{54x^3 + 108}{x^3}$

P.O.I $\Rightarrow x = -\sqrt[3]{2}$

f'' up $-\sqrt[3]{2}$ down 0 up

+ + + + $-\sqrt[3]{2}$ – – – – 0 + + + + + +

V.A. $\Rightarrow x = 0$

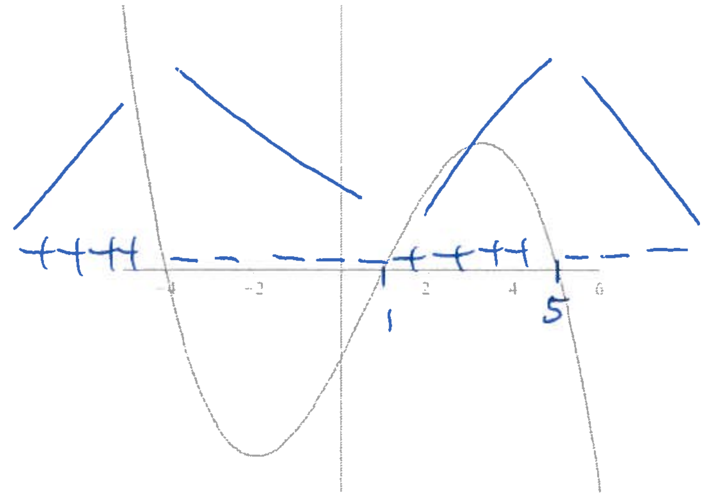
c) $f(x)$ has a vertical asymptote at $x = 54$. $\Rightarrow x = 0$

d) $f(x)$ has a local minimum at the point $(1, 77)$.

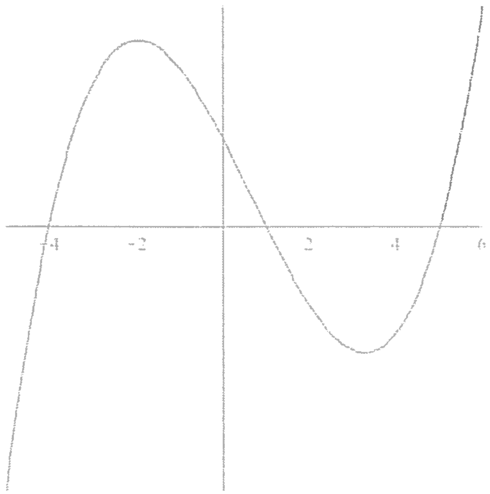
e) $f(x)$ is increasing on the interval $(-\infty, 0)$. $\Rightarrow (1, \infty)$

Question 14

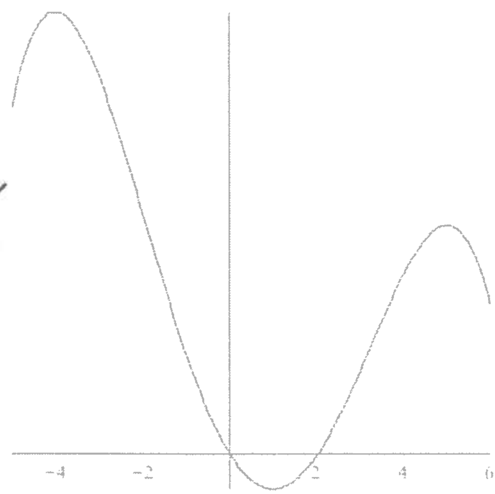
The graph of $f'(x)$ is shown below. Which of the following could represent the graph of $f(x)$?



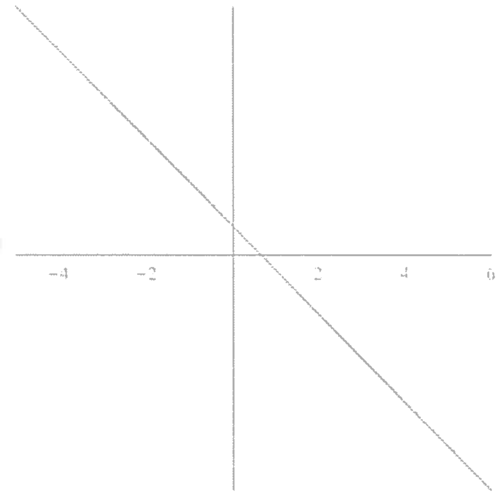
a)



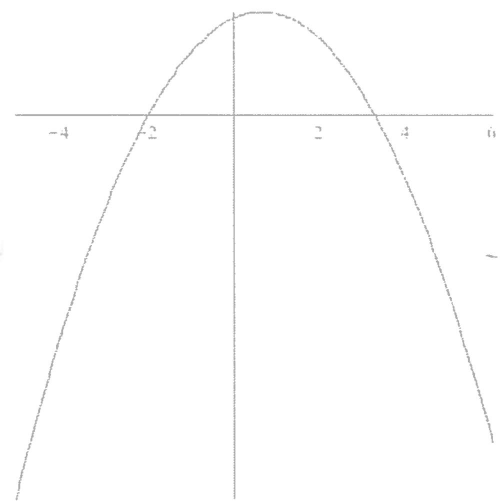
b)



c)



d)



Question 15

Determine whether or not the given function is one-to-one and, if so, find the inverse. If $f(x) = 6x - 2$ has an inverse, give the domain of f^{-1}.

$f'(x) = 6 > 0 \Rightarrow$ Monotone $\Rightarrow$ 1-1.

(1) $y = f(x) = 6x - 2$.

(2) $x = 6y - 2$

(3) $x + 2 = 6y$

$y = \frac{x+2}{6}$

$f^{-1}(x) = \frac{x+2}{6}$ and $D(f^{-1}) = \mathbb{R}$

a) Not one-to-one.

b) $f^{-1}(x) = 6x - 2$; domain: $(-\infty, -2)$

c) $f^{-1}(x) = \frac{1}{6}x + \frac{1}{3}$; domain: $(-\infty, \infty)$

d) $f^{-1}(x) = 6x - 2$; domain: $(-\infty, \infty)$

e) $f^{-1}(x) = -\frac{1}{6}x - \frac{1}{3}$; domain: $(-2, \infty)$

Question 16

Suppose that f has an inverse and $f(\underset{a}{-2}) = \underset{b}{3}$, $f'(-2) = \frac{6}{7}$. What is $(f^{-1})'(3)$?

$f^{-1}(3) = -2$

and $(f^{-1})'(3) = \frac{1}{f'(-2)} = \frac{1}{\frac{6}{7}} = \frac{7}{6}$

a) $\frac{7}{6}$

b) $\frac{13}{6}$

c) $-\frac{6}{7}$

d) $\frac{6}{7}$

e) $\frac{7}{3}$

Question 17

Suppose that $f(x) = 3x^3 + 6$ is differentiable and has an inverse and $f(\underset{a}{4}) = \underset{b}{198}$. Find $(f^{-1})'(198)$. $\Rightarrow f^{-1}(198) = 4$ and.

$f'(x) = 9x^2$, $f'(4) = 144$

$(f^{-1})'(198) = \frac{1}{f'(4)} = \frac{1}{144}$.

a) $\frac{1}{72}$

b) $-\frac{1}{144}$

c) 288

d) 144

e) $\frac{1}{144}$

Question 18

Suppose that $f(x) = 2x + 2\pi + \cos(x)$ is differentiable and has an inverse for $0 < x < 2\pi$ and $f(1\pi) = 4\pi - 1$. Find $(f^{-1})'(4\pi - 1)$.

$f'(x) = 2 - \sin(x)$ and $f^{-1}(4\pi - 1) = \pi$.

$f'(\pi) = 2 - 0 = 2$, Then

$(f^{-1})'(4\pi - 1) = \frac{1}{f'(\pi)} = \frac{1}{2}$

a) -1

b) $\frac{1}{2}$

c) $\frac{1}{4}$

d) $-\frac{1}{2}$

e) 1

Question 19

Differentiate: $y = 4xe^{2x^3}$ (Product)

$\Rightarrow y' = 4e^{2x^3} + 4x\underbrace{(2x^3)'}_{6x^2} e^{2x^3}$

$= 4e^{2x^3} + 24x^3 e^{2x^3}$

a) $y' = 4e^{2x^3} + 4xe^{2x^3}$

b) $y' = 4e^{2x^3} - 24x^3e^{2x^3}$

c) $y' = 4e^{2x^3}$

d) $y' = 4e^{6x^2}$

e) $y' = e^{2x^3} + 6x^3e^{2x^3}$

Question 20

Differentiate: $y = \ln(2x^2 + 3)$

$y' = \dfrac{(2x^2+3)'}{2x^2+3}$

$= \dfrac{4x}{2x^2+3}$

a) $y' = -\dfrac{4x}{(2x^2+3)^2}$

b) $y' = \dfrac{2}{2x^2+3}$

c) $y' = \dfrac{4x}{2x^2+3}$

d) $y' = -\dfrac{1}{(2x^2+3)^2}$

e) $y' = \dfrac{1}{2x^2-3}$