PRINTABLE VERSION

Sol

Practice Final

Question 1

Evaluate the limit: $\lim_{x\to 2}\left(\frac{5x^3-40}{x-2}\right)$ $\left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x\to 2} \frac{15x^2}{1} = 15\cdot 2^2 = 60$

a) $\frac{1}{60}$

b) 0

c) does not exist

d) -60

e) 60

Question 2

$$\lim_{h\to 0}\frac{1-\frac{64}{h^2}}{1+\frac{64}{h^2}} = \lim_{h\to 0}\frac{h^2\left(1-\frac{64}{h^2}\right)}{h^2\left(1+\frac{64}{h^2}\right)} = \lim_{h\to 0}\frac{h^2-64}{h^2+64} = \frac{-64}{64} = -1$$

a) -8

b) -1

c) does not exist

d) 0

e) 1

Q3

① $\lim_{x\to a^+} f(x)$, ② $\lim_{x\to a^-} f(x)$, ③ $f(a)$.

$x=0$; ① $=0$, ② $=-6$, ③ $=-6$, ① $\neq$ ② jump disconti.

$x=1$; ① $=1$ ② $=1$ ③ $=1$, ① $=$ ② $=$ ③ conti.

$x=2$; ① $=2$ ② $=1$ ③ $=2$ ① $\neq$ ② jump. disconti.

Question 3

Classify the discontinuities (if any) for the given function

$$f(x)=\begin{cases}-6 & x\le 0\\ x^2 & 0<x<1\\ 1 & 1\le x<2\\ x & x\ge 2\end{cases}$$

a) The function has a removable discontinuity at $x = 2$

b) The function has a jump discontinuity at $x = 0$ and 2

c) The function is continuous for all x

d) The function has an infinite discontinuity at $x = 2$

e) The function has a removable discontinuity at $x = 1$

① $\lim_{x\to a^+} f(x)$ ② $\lim_{x\to a^-} f(x)$ ③ $f(a)$

Question 4

Give the values of A and B for the function $f(x)$ to be continuous at both $x = 1$ and $x = 6$.

$$f(x) = \begin{cases} Ax - B & x < 1 \\ -10x & 1 \le x < 6 \\ Bx^2 - A & x \ge 6 \end{cases}$$

$x=1$.
① $= -10$
② $= A-B$
③ $= A-B$
$\Downarrow$
① = ② = ③
$\Rightarrow A-B=-10$ —(1)

$x=6$
① $= 36B-A$
② $= -60$
③ $= 36B-A$
$\Downarrow$
① = ② = ③
$36B-A=-60$ —(2)

$\begin{cases} A-B=-10 & \text{—(1)} \\ 36B-A=-60 & \text{—(2)} \end{cases}$

$\Rightarrow B=-2$
$A=-12$.

a) $A = -11$ and $B = -2$

b) $A = -13$ and $B = -2$

c) $A = -12$ and $B = -3$

d) $A = -12$ and $B = -1$

e) $A = -12$ and $B = -2$

Question 5

Given that $f(x) = \dfrac{\sqrt{x+5} - 4}{x - 11}$, define the function $f(x)$ at 11 so that it becomes continuous at 11.

Check $\lim_{x\to 11} \dfrac{\sqrt{x+5}-4}{x-11}$ $\left(\frac{0}{0}\right)$ $\overset{L'}{=}$ $\lim_{x\to 11} \dfrac{\frac{1}{2\sqrt{x+5}}}{1} = \dfrac{\frac{1}{2\cdot 4}}{1} = \dfrac{1}{8}$

a) $f(11) = 5$

b) $f(11) = \frac{1}{8}$

Now $f(11)$ DNE, so if we define $f(11)=\frac{1}{8}$ then f is conti. everywhere

c) Not possible because there is an infinite discontinuity at the given point.

d) $f(11) = 8$

e) $f(11) = 0$

Question 6

$$\lim_{x\to 0}\frac{5x}{\cot(6x)} = \lim_{x\to 0}\frac{5x}{\cos(6x)}\cdot\sin(6x) = \frac{0}{1} = 0$$

a) $\frac{6}{5}$

b) 0

c) The limit does not exist.

d) $\frac{5}{6}$

e) 1

Question 7

Given $f(x) = 7x^2 - 3\sqrt{x}$ which of the following expressions will represent $f'(x)$?

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{(7(x+h)^2-3\sqrt{x+h})-(7x^2-3\sqrt{x})}{h}$$

a) $\lim_{h\to 0}\frac{7(x+h)^2-3\sqrt{x+h}}{h}$

b) $\lim_{h\to 0}\frac{(7x^2-3\sqrt{x}+h)-(7x^2-3\sqrt{x})}{h}$

c) $\frac{\left(7(x+h)^2-3\sqrt{x+h}\right)-(7x^2-3\sqrt{x})}{h}$

d) $\lim_{h\to 0}\frac{\left(7(x+h)^2-3\sqrt{x}+h\right)-(7x^2-3\sqrt{x})}{h}$

e) $\lim_{h\to x}\frac{\left(7(x+h)^2-3\sqrt{x+h}\right)-(7x^2-3\sqrt{x})}{h}$

Question 8

Determine the values of the constants B and C so that the function given below is differentiable.

$$f(x) = \begin{cases} 9x^3 & x < 1 \\ Bx + C & x \geq 1 \end{cases}$$

I. conti. and II. $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$ exists.

I. Continuity
At $x=1$, ① $=B+C$
② $=9$
③ $=9$
$\Downarrow$
① = ② = ③
$B+C=9$ —(1)

II. $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$ exists

implies $\underbrace{\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}}_{\parallel\ B} = \underbrace{\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}}_{\parallel\ 27\cdot 1^2=27}$

$\Rightarrow B=27$.

since (1), $\Rightarrow C=-18$

a) $\{B = 27, C = 54\}$

b) $\{B = 27, C = 63\}$

c) $\{B = 27, C = -18\}$

d) $\{B = -54, C = -18\}$

e) $\{B = -27, C = 36\}$

Question 9

For $g(x) = x + 3\sin(x) + \tan(x)$, find $g'\left(\frac{\pi}{6}\right)$.

$g'(x) = 1 + 3\cos(x) + \sec^2(x)$

$g'(\frac{\pi}{6}) = (1 + 3\cos(\frac{\pi}{6}) + \sec^2(\frac{\pi}{6}))$

$= 1 + 3\cdot\frac{\sqrt{3}}{2} + \left(\frac{2}{\sqrt{3}}\right)^2$

$= 1 + \frac{3\sqrt{3}}{2} + \frac{4}{3} = \frac{7}{3} + \frac{3}{2}\sqrt{3}$.

a) $2 - \sqrt{3}$

b) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

c) $2 + \sqrt{3}$

d) $\frac{7}{3} + \frac{3\sqrt{3}}{2} + \frac{\pi}{6}$

e) $\frac{7}{3} + \frac{3\sqrt{3}}{2}$

Question 10

Calculate the derivative of the given function $f(x) = 2\sin^5(\sqrt{x})$ chain rule

$= 2(\sin(\sqrt{x}))^5$

$f'(x) = 10(\sin(\sqrt{x}))^4 \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{5\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

a) $f'(x) = \frac{5\cos(\sqrt{x})}{\sqrt{x}}$

b) $f'(x) = 10\cos(\sqrt{x})$

c) $f'(x) = \frac{5\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

d) $f'(x) = \frac{20\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

e) $f'(x) = 10\sin^4(\sqrt{x})\cos(\sqrt{x})$

Question 11 ① slope ② point.

Find the equation of the tangent line for $f(x) = 2\tan(x)$ at $x = \frac{\pi}{4}$

① $f'(x) = 2\sec^2(x)$.

② Point, $(\frac{\pi}{4}, f(\frac{\pi}{4})) = (\frac{\pi}{4}, 2)$.

slope: $f'(\frac{\pi}{4}) = 2\cdot\left(\frac{2}{\sqrt{2}}\right)^2 = 2\cdot\frac{4}{2} = 4$ $\Rightarrow$ line: $y - 2 = 4(x - \frac{\pi}{4})$.

a) $y = 4\left(x - \frac{\pi}{4}\right)$

b) $y = 4\left(x - \frac{\pi}{4}\right) + 2$

c) $y = \left(x - \frac{\pi}{4}\right) + 2\sqrt{2}$

d) $y = 2\left(x - \frac{\pi}{4}\right) + 4$

e) $y = 2\left(x - \frac{\pi}{4}\right) + 2\sqrt{2}$

Tangent line: ① Slope, ② point.

① $\frac{d}{dx}\left(x^2+xy+2y^2=18\right) \Rightarrow 2x+y+x\frac{dy}{dx}+4y\frac{dy}{dx}=0$

$\Rightarrow (x+4y)\frac{dy}{dx} = -2x-y \Rightarrow \frac{dy}{dx} = \frac{-2x-y}{x+4y}$

slope @ $x=3, y=-3$, $\left.\frac{dy}{dx}\right|_{(3,-3)} = \frac{-3}{-9} = \frac{1}{3}$

② point: $(3,-3)$

$\Rightarrow$ line eq. $y+3 = \frac{1}{3}(x-3)$

Question 12

Find the equation of the tangent line to the curve at the point $(3, -3)$ given $x^2 + xy + 2y^2 = 18$.

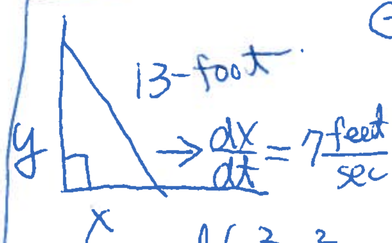
a) $y = -\frac{1}{3}x + 4$

b) $y = \frac{1}{3}x - 4$

c) $y = -\frac{1}{3}x - 9$

d) $y = 3x + 12$

e) $y = -3x - 12$



Given $x^2+y^2=13^2$, $\frac{dx}{dt}=7$.

Find $\frac{d(xy)}{dt}$ as $x=12$.

$\frac{d}{dt}\left(x^2+y^2=13\right) \Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

At $x=12$, $y=5$. $2\cdot12\cdot7+2\cdot5\frac{dy}{dt}=0 \Rightarrow \frac{dy}{dt}=\frac{-84}{5}$

Question 13

A 13-foot ladder is leaning against a vertical wall. If the bottom of the ladder is being pulled away from the wall at the rate of 7 feet per second, at what rate is the area of the triangle formed by the wall, the ground, and the ladder changing, in square feet per second, at the instant the bottom of the ladder is 12 feet from the wall?

Then $\frac{d(xy)}{dt} = y\cdot\frac{dx}{dt} + x\frac{dy}{dt}$ at $(12,5)$

$= 5\cdot7 + 12\cdot\frac{-84}{5}$

$= 35 - \frac{1008}{5} = \frac{-833}{5}$

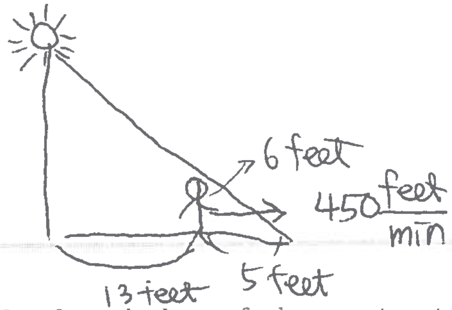
a) $\frac{833}{10}$

b) $-\frac{833}{20}$

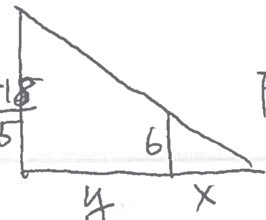
c) $-\frac{833}{5}$

d) $-\frac{833}{10}$

e) $\frac{833}{5}$



$\Rightarrow$



Find $\frac{dx}{dt}$ as $\frac{dy}{dt} = 450$.

Question 14

A man standing 13 feet from the base of a lamppost casts a shadow 5 feet long. If the man is 6 feet tall and walks away from the lamppost at a speed of 450 feet per minute, at what rate, in feet per minute, will the length of his shadow be changing?

The height of lamppost: $\frac{h}{13+5} = \frac{6}{5} \Rightarrow h = \frac{6}{5} \cdot 18$.

a) $-\frac{4500}{13}$

b) $\frac{1125}{13}$

c) $\frac{4500}{13}$

d) $\frac{2250}{13}$

e) $-\frac{2250}{13}$

$\Rightarrow \frac{\frac{6\cdot 18}{5}}{x+y} = \frac{6}{x} \Rightarrow \frac{18}{5}x = x+y \Rightarrow \frac{13}{5}x = y$.

do $\frac{d}{dt}$

$\frac{13}{5}\frac{dx}{dt} = \frac{dy}{dt} \Rightarrow \frac{dx}{dt} = \frac{5}{13}\cdot\frac{dy}{dt} = \frac{5}{13}\cdot 450 = \frac{2250}{13}$

Question 15

Determine if the function $f(x) = 3\sqrt{9 - x^2}$ satisfies the Mean Value Theorem on $[0, 3]$. If so, find all numbers c on the interval that satisfy the theorem.

a = 0, b = 3

Find c s.t. $f'(c) = \frac{f(b) - f(a)}{b-a}$.

a) $c = \frac{-3\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$

$f'(x) = \frac{3\cdot(-2x)}{2\sqrt{9-x^2}}$ and $f(a) = 3\cdot\sqrt{9} = 9$.

$f(b) = f(3) = 0$.

b) The Mean Value Theorem does not apply to this function on the given interval.

c) $c = \frac{3\sqrt{2}}{2}$

d) $c = \frac{-3\sqrt{2}}{2}$

e) $c = 0$

$\Rightarrow \frac{-6c}{2\sqrt{9-c^2}} = f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{0-9}{3-0} = -3$

$\Rightarrow \frac{-6c}{2\sqrt{9-c^2}} = \frac{-3}{1} \Rightarrow 2c = 2\sqrt{9-c^2}$

$\Rightarrow (2c)^2 = (2\sqrt{9-c^2})^2$

$\Rightarrow 4c^2 = 4(9-c^2) \Rightarrow 8c^2 = 36$

$\Rightarrow c = +\frac{3\sqrt{2}}{2}$ ~~or $-\frac{3\sqrt{2}}{2}$~~

$c \in (0,3)$

Question 16

Find the intervals on which $f(x) = \frac{4x}{x^2+36}$ decreases. $D(f)$: $x \in \mathbb{R}$.

$f'(x) = \frac{4(x^2+36) - 2x\cdot 4x}{(x^2+36)^2} = \frac{-4x^2+4\cdot 36}{(x^2+36)^2} = 0$

a) $(-6, 6)$

$\Rightarrow -4x^2 + 4\cdot 36 = 0 \Rightarrow x = 6$ or -6.

$- - - \; -6 \; + + + + + \; 6 \; - - - \Rightarrow (-\infty, -6) \cup (6, \infty)$.

b) $(-\infty, \infty)$

c) $(6, \infty)$

d) $(-\infty, -6) \cup (0, 6)$

e) $(-\infty, -6) \cup (6, \infty)$

$f'(x) = 2x - 16. \Rightarrow f'(x) = 0,\ x = 8, \Rightarrow$ critical numbe

f' $- - - -\ 8\ + + + +$ $\Rightarrow$ local min

$f(8) = 64 - 128 + 5 = -59 \rightarrow$ abs min

$f(0) = 5 \rightarrow$ abs max.

$f(10) = 100 - 160 + 5 = -55$

Question 17

Find the critical numbers of $f(x) = x^2 - 16x + 5$ and classify all extreme values given $0 \leq x \leq 10$.

a) Critical no. 8 and 10; local max $f(10) = f(8) = -55$.

b) Critical no. 0; local max $f(0) = 5$.

c) Critical no. 8; absolute max $f(0) = 5$; local and absolute min $f(8) = -59$.

d) Critical nos. 0 and 8; local and absolute min $f(8) = -59$; absolute max $f(10) = -55$.

e) No critical numbers, no extreme values.

$f'(x) = \frac{5}{3}(x+2)^{\frac{2}{3}}$, $f''(x) = \frac{5}{3} \cdot \frac{2}{3}(x+2)^{-\frac{1}{3}} = \frac{10}{9} \frac{1}{(x+2)^{\frac{1}{3}}}$

$\Rightarrow f''(x)$ DNE implies $x = -2$.

Question 18

Describe the concavity of the graph of $f(x) = (x+2)^{5/3}$ and find the points of inflection (if any).

f'' up | down

$+ + + +\ -2\ - - - -$

a) concave up on $(-\infty, 0)$); concave down on $(0, \infty)$; pt of inflection $(0, 0$.

b) concave down on $(-\infty, -2)$; concave up on $(-2, \infty)$; pt of inflection $(-2, 0)$.

c) concave up on $(-\infty, \infty)$; no points of inflection

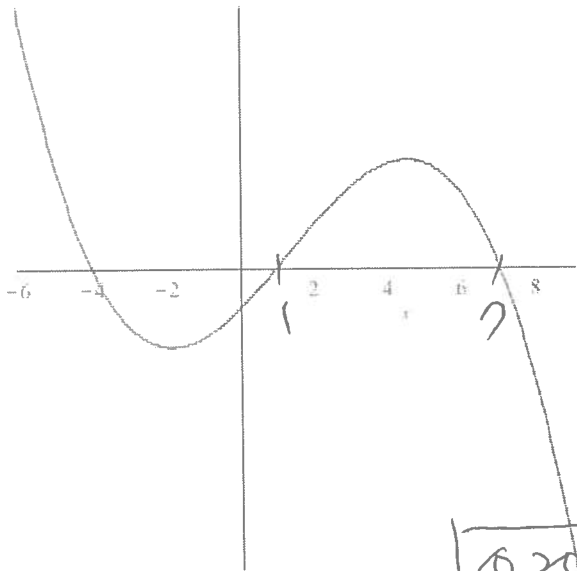
d) concave up on $(-\infty, 2)$; concave down on $(2, \infty)$; pt of inflection $(2, 0)$.

e) concave down on $(-\infty, \infty)$; no points of inflection

Question 19

Given the graph of $f'(x)$ below, where is $f(x)$ decreasing? $\Rightarrow f'(x) < 0$

$(-4, 1) \cup (7, \infty)$



Q 20

① Vertical asym:

$f(x) \to \infty$ implies $3x+4=0$

$\Rightarrow x = -\frac{4}{3}$

② horizontal asym.

$x \to \infty$. $y \to \frac{2}{3}$.

a) $f(x)$ is decreasing on the interval $(-4, \infty)$.

b) $f(x)$ is decreasing on the intervals $(-\infty, -4)$ and $(1, 7)$.

c) $f(x)$ is decreasing on the intervals $(-4, 1)$ and $(7, \infty)$.

d) $f(x)$ is decreasing on the interval $(-4, 7)$.

e) $f(x)$ is decreasing on the interval $(-\infty, 7)$.

Question 20

Find the vertical and horizontal asymptotes of $f(x) = \dfrac{2x}{3x+4}$.

a) vertical asymptote: $x = -\dfrac{4}{3}$; no horizontal asymptote.

b) vertical asymptote: $x = -\dfrac{4}{3}$; horizontal asymptote: $y = \dfrac{2}{3}$.

c) vertical asymptote: $x = -\dfrac{4}{3}$; horizontal asymptote: $y = 0$.

d) vertical asymptote: $x = \dfrac{2}{3}$; horizontal asymptote: $y = -\dfrac{4}{3}$.

e) no vertical asymptote; horizontal asymptote: $y = \dfrac{2}{3}$.

$(f^{-1})'(b) = \frac{1}{f'(a)}$

Question 21

Suppose that $f(x) = \frac{x+6}{x-1}$ is differentiable and has an inverse for $x > 1$ and $f(\underset{a}{3}) = \underset{b}{\frac{9}{2}}$. Find $(f^{-1})'\left(\frac{9}{2}\right)$.

$f'(x) = \frac{x-1-(x+6)}{(x-1)^2} = \frac{-7}{(x-1)^2}$, $f'(3) = \frac{-7}{4}$

$(f^{-1})'\left(\frac{9}{2}\right) = \frac{1}{f'(3)} = -\frac{4}{7}$

a) $\frac{4}{7}$

b) $-\frac{2}{7}$

c) $-\frac{8}{7}$

d) $\frac{8}{7}$

e) $-\frac{4}{7}$

Question 22

Differentiate: $y = \underline{2x e^{4x^2}}$ product

$y' = 2e^{4x^2} + 2x \cdot (4x^2)' e^{4x^2}$
$= 2e^{4x^2} + 16x^2 e^{4x^2}$

a) $y' = e^{4x^2} + 8x^2e^{4x^2}$

b) $y' = 2e^{4x^2}$

c) $y' = 2e^{4x^2} + 2xe^{4x^2}$

d) $y' = 2e^{8x}$

e) $y' = 2e^{4x^2} + 16x^2e^{4x^2}$

$\ln g(x) = \ln\left((x^2+1)^2(x-1)^7x^3\right)$
$= 2\ln(x^2+1) + 7\ln(x-1) + 3\ln(x)$

derivative

$\frac{g'(x)}{g(x)} = \frac{2(2x)}{x^2+1} + \frac{7}{x-1} + \frac{3}{x}$

Question 23

Calculate the derivative by logarithmic differentiation: $g(x) = (x^2+1)^2(x-1)^7x^3$.

$g'(x) = \left[\frac{4x}{x^2+1} + \frac{7}{x-1} + \frac{3}{x}\right](x^2+1)^2(x-1)^7x^3$

a) $g'(x) = \left(x^2+1\right)^2(x-1)^7x^3\left(\frac{2x}{x^2+1} + \frac{1}{x-1} + \frac{3}{x}\right)$

b) $g'(x) = \left(x^2+1\right)^2(x-1)^7x^3\left(\frac{4x}{x^2+1} - \frac{7}{x-1} - \frac{3}{x}\right)$

c) $g'(x) = \frac{4x}{x^2+1} - \frac{7}{x-1} - \frac{3}{x}$

d) $g'(x) = \left(x^2+1\right)^2 (x-1)^7 x^3 \left(\frac{4x}{x^2+1} + \frac{7}{x-1} + \frac{3}{x}\right)$

e) $g'(x) = \frac{4x}{x^2+1} + \frac{7}{x-1} + \frac{3}{x}$

Question 24

Differentiate $y = \arcsin\left(\frac{e^{5x}}{2}\right)$.

$$y' = \frac{1}{\sqrt{1-\left(\frac{e^{5x}}{2}\right)^2}} \cdot \left(\frac{e^{5x}}{2}\right)'$$

$$= \frac{5}{2} e^{5x} \cdot \frac{1}{\sqrt{1-\left(\frac{e^{5x}}{2}\right)^2}}$$

$$= 5e^{5x} \frac{1}{\sqrt{4-4\left(\frac{e^{5x}}{2}\right)^2}} = \frac{5e^{5x}}{\sqrt{4-e^{10x}}}.$$

a) $\frac{10e^{5x}}{\sqrt{4-e^{10x}}}$

b) $\frac{5e^{5x}}{\sqrt{4+e^{10x}}}$

c) $\frac{5e^{5x}}{\sqrt{4-e^{10x}}}$

d) $\frac{5e^{5x}}{4+e^{10x}}$

e) $\frac{10e^{5x}}{4+e^{10x}}$

Question 25

Differentiate the given function $y = (\cosh(7x))^x$.

$$\ln y = x \cdot \ln(\cosh(7x)).$$

$$y' = \left[\ln(\cosh(7x)) + x \cdot \frac{7 \cdot \sinh(7x)}{\cosh(7x)}\right](\cosh(7x))^x$$

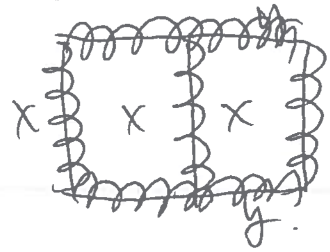
a) $(\cosh(7x))^x \left(\ln(\cosh(7x)) + \frac{7}{\cosh(7x)}\right)$

b) $(\sinh(7x))^x \left(\ln(\cosh(7x)) + \frac{7}{\cosh(7x)}\right)$

c) $(\cosh(7x))^x \left(\ln(\cosh(7x)) + \frac{7\sinh(7x)}{\cosh(7x)}\right)$

d) $(\cosh(7x))^x \left(\ln(\cosh(7x)) + \frac{7x\sinh(7x)}{\cosh(7x)}\right)$

e) $(\sinh(7x))^x \left(\ln(\sinh(7x)) + \dfrac{7x\cosh(7x)}{\sinh(7x)} \right)$



Question 26

A rectangular playground is to be fenced off and divided into two parts by a fence parallel to one side of the playground. 480 feet of fencing is used. Find the dimensions of the playground that will enclose the greatest total area.

Max ① function: xy.

② Condition: $3x+2y=480$.

$y=\dfrac{480-3x}{2}$.

③ $x>0,\ y>0$.

④ $f(x)=xy=x\cdot\dfrac{480-3x}{2}=\dfrac{480x-3x^2}{2}$

$=240x-\frac{3}{2}x^2$.

$f'(x)=240-3x$

$f'(x)=0$ implies $x=80$.

0   80 $\Rightarrow$ MAX

$y=120$.

$\Rightarrow$ 120 by 80 with 80.

a) 130 by 90 feet with the divider 130 feet long

b) 115 by 85 feet with the divider 116 feet long

c) 120 by 120 feet with the divider 120 feet long

d) 120 by 80 feet with the divider 80 feet long

e) 140 by 90 feet with the divider 90 feet long

Question 27

Use differentials to estimate the value $(15.8)^{1/4}$.

① $f(x)=x^{\frac{1}{4}}$, $f'(x)=\frac{1}{4}x^{-\frac{3}{4}}$

$a+h=15.8$.

② $a=16$.

③ $h=-0.2$

④ $(15.8)^{\frac{1}{4}}=f(15.8)\approx f(16)+f'(16)\cdot h$

$=16^{\frac{1}{4}}+\frac{1}{4}\frac{1}{(16)^{\frac{3}{4}}}\cdot\left(-\frac{2}{10}\right)=2+\frac{1}{\cancel{32}\,16}\cdot\left(-\frac{2}{10}\right)$

$=\dfrac{319}{160}$

a) $\dfrac{321}{160}$

b) $\dfrac{399}{160}$

c) $\dfrac{9}{5}$

d) $\dfrac{319}{160}$

e) $\dfrac{239}{160}$

Question 28

Calculate the limit: $\lim\limits_{x\to\infty}\left(\cos\left(\frac{3}{x}\right)\right)^x$. $\ \frac{1^\infty}{}$

$\lim\limits_{x\to\infty} e^{\ln(\cos(\frac{3}{x}))^x} = \lim\limits_{x\to\infty} e^{x\ln(\cos(\frac{3}{x}))}$

$= e^{\lim\limits_{x\to\infty} x\ln(\cos(\frac{3}{x}))} = e^0 = 1.$

exp fn. is conti. &

$\lim\limits_{x\to\infty} x\cdot\ln(\cos(\frac{3}{x}))$

$=\lim\limits_{x\to\infty}\dfrac{\ln(\cos(\frac{3}{x}))}{\frac{1}{x}}$

$\left(\frac{0}{0}\right) = \lim\limits_{x\to\infty}\dfrac{\frac{-\sin(\frac{3}{x})}{\cos(\frac{3}{x})}\cdot\frac{-3}{x^2}}{-\frac{1}{x^2}} = \lim\limits_{x\to\infty} -3\,\dfrac{\sin(\frac{3}{x})}{\cos(\frac{3}{x})} = 0$

a) e^3

b) 3

c) 0

d) 1

e) -3

Question 29

$\Rightarrow$ Max.

Compute the upper Riemann sum for the given function $f(x) = x^2$ over the interval $x \in [-1, 1]$ with respect to the partition $P = \left[-1, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, 1\right]$.

a) $\frac{21}{16}$

b) $\frac{91}{96}$

c) $\frac{49}{64}$

d) $\frac{7}{32}$

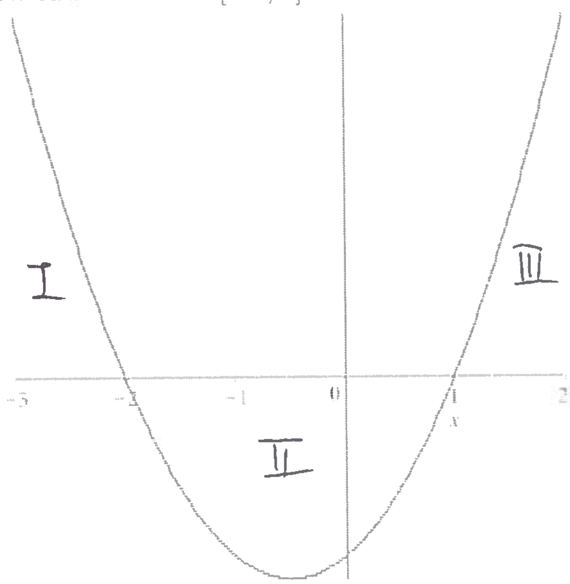
e) $\frac{7}{12}$

Subinterval	length	Max of f(x)
$[-1, -\frac{1}{4}]$	$\frac{3}{4}$	$f(-1) = 1$
$[-\frac{1}{4}, \frac{1}{4}]$	$\frac{1}{2}$	$f(\frac{1}{4}) = \frac{1}{16}$
$[\frac{1}{4}, \frac{3}{4}]$	$\frac{1}{2}$	$f(\frac{3}{4}) = \frac{9}{16}$
$[\frac{3}{4}, 1]$	$\frac{1}{4}$	$f(1) = 1$

Upper sum $= U_f = \frac{3}{4} \cdot 1 + \frac{1}{2} \cdot \frac{1}{16} + \frac{1}{2} \cdot \frac{9}{16} + \frac{1}{4} \cdot 1 = \frac{42}{32} = \frac{21}{16}$

Question 30

The graph of f is shown below on the interval $[-3, 2]$.



$\mathrm{I} = \frac{11}{6}$

$\mathrm{II} = \frac{9}{2}$

$\mathrm{III} = \frac{11}{6}$

The area bounded between the graph of f and the x-axis on $[-3, -2]$ is $\frac{11}{6}$, the area bounded between the graph of f and the x-axis on $[-2, 1]$ is $\frac{9}{2}$, and the area bounded between the graph of f

and the x-axis on $[1, 2]$ is $\frac{11}{6}$. Determine $\int_{-3}^{2} f(x)\, dx$. $= \mathrm{I} - \mathrm{II} + \mathrm{IV}$

$= \frac{11}{6} - \frac{9}{2} + \frac{11}{6}$

$= -\frac{5}{6}$

a) $-\frac{11}{6}$

b) $\frac{19}{3}$

c) $-\frac{5}{6}$

d) 0

e) $\frac{49}{6}$

Question 31

Find the derivate of $F(x) = \int_0^{x\sin(x)} \sqrt{25 - t^2}\, dx$. $F'(x) = (x\sin(x))' \cdot \sqrt{25 - (x\sin(x))^2}$

$= \left[\sin(x) + x\cos(x)\right]\sqrt{25 - x^2\sin^2(x)}$.

a) $\sqrt{25 - x^2(\sin(x))^2}$

b) $(\sin(x) + x\cos(x))\sqrt{25 - x^2(\sin(x))^2}$

c) $(\sin(x) + x\cos(x))\sqrt{25 - x^2}$

d) $-\dfrac{x\sin(x)}{\sqrt{25 - x^2(\sin(x))^2}}$

e) $\sqrt{25 - x^2}$

Question 32

Evaluate the definite integral: $\int_1^2 5\,x\left(x^2 + 1\right) dx$

let $u = x^2+1$, $du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$

$\int_{1^2+1}^{2^2+1} 5 \cdot u \frac{du}{2} = \frac{5}{2}\int_2^5 u\, du = \frac{5}{2} \cdot \frac{u^2}{2}\Big|_2^5$

a) $\frac{50}{3}$

b) $\frac{105}{4}$

c) $\frac{45}{4}$

$= \frac{5}{4}\left[5^2 - 2^2\right] = \frac{5}{4} \cdot 21 = \frac{105}{4}$

d) 40

e) $\frac{128}{3}$

Question 33

Calculate the indefinite integral: $\int \left(4\cosh(x) + x^3\right) dx$.

$$4\sinh(x) + \frac{x^4}{4} + C.$$

a) $4\sinh(x) + \frac{1}{4}x^4 + C$

b) $4\sinh(x) + \frac{3}{4}x^4 + C$

c) $-4\sinh(x) - \frac{1}{4}x^4 + C$

d) $4\sinh(x) + \frac{1}{3}x^3 + C$

e) $4\sinh(x) + 3x^2 + C$

Question 34

Calculate: $\int_{-1}^{0} 9\,x^2\left(2x^3+2\right)^2 dx$

let $u = 2x^3+2$. $du = 6x^2 dx \Rightarrow \frac{du}{6} = x^2 dx$

$$\int_{2(-1)^3+2}^{2\cdot 0^3+2} 9 \cdot u^2 \frac{du}{6} = \frac{9}{6}\int_0^2 u^2 du$$

$$= \frac{9}{6} \cdot \frac{u^3}{3}\Big|_0^2$$

$$= \frac{3}{2} \cdot \frac{1}{3}\left[2^3 - 0^3\right] = \frac{24}{6} = 4$$

a) 4

b) $\frac{4}{3}$

c) 8

d) $\frac{2}{3}$

e) $\frac{7}{3}$