One-to-One: (1) Use horizontal line test.
(2) Check first derivative of f.
if f is monotone (i.e. $f' \geq 0$ on $D(f)$ or $f' \leq 0$ on $D(f)$)
Then f is One-to One

Math 1431, Section 17699

Homework 9 (10 points)

Due 4/2 in Recitation

Name: Sol PSID:

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

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1. (Section 4.1, Problem 14) Given $f(x) = x^3 - 6x^2 + 12x$

$D(f) = \mathbb{R}$. and $f'(x) = 3x^2 - 12x + 12$
$= 3(x^2 - 4x + 4) = 3(x-2)^2 \geq 0$

$\Rightarrow$ f is always increasing on its domain $\mathbb{R}$.

$\Rightarrow$ f is invertible.

2. (Section 4.1, Problem 20) Given $f(x) = \frac{x+2}{x}$, $D(f) = \{x \neq 0\}$.

1. $f'(x) = \frac{x - (x+2)}{x^2} = \frac{-2}{x^2} \leq 0$ on $D(f)$.

$\Rightarrow$ f is always decreasing on $D(f)$.

$\Rightarrow$ f is invertible.

2. Find f^{-1}, (1) let $y = f(x) = \frac{x+2}{x}$ (2) switch x & y $x = \frac{y+2}{y} = \frac{y}{y} + \frac{2}{y} = 1 + \frac{2}{y}$

(3) Solve y: $x - 1 = \frac{2}{y} \Rightarrow y = \frac{2}{x-1}$.

3. $D(f^{-1}) = \{x \neq 1\}$

Find inverse function of an invertible function
Given $f(x)$, then
(1) Let $y = f(x)$, (2) Switch x and y, (3) solve y.
Thus, we got $f^{-1}(x) = y$.

3. (Section 4.1, Problem 28) Given $f(x) = \sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

Find $(f^{-1})'(\frac{1}{2})$: let $c = f^{-1}(\frac{1}{2}) \Rightarrow f(c) = \frac{1}{2}$

Using $(f^{-1})'(\frac{1}{2}) = \frac{1}{f'(c)}$, we need to know "c"

Such that $\sin(c) = \frac{1}{2} \Rightarrow c = \frac{\pi}{6}$

and $f'(x) = \cos x$, Then,

$$(f^{-1})'(\tfrac{1}{2}) = \frac{1}{\cos(\frac{\pi}{6})} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}.$$

4. (Section 4.1, Problem 33) Given $f(x) = \frac{1}{3}x^3 - 8x^2 + kx$.

To Find k s.t. f is one-to-one, it is sufficiently to check f' s.t. f is monotone.

Then $f'(x) = x^2 - 16x + k = (x^2 - 16x + 64) - 64 + k$
$= (x-8)^2 - 64 + k$.

$\Rightarrow -64 + k \geq 0 \Rightarrow k \geq 64$,

5. (Section 4.2, Problem 12) Given $f(x) = 7^{\sin x}$.

Take "ln" on both sides, we have

$\ln f(x) = \ln 7^{\sin x} = \sin(x)(\ln 7)$.

Do derivative on both sides, we have

$$\frac{f'(x)}{f(x)} = (\ln 7) \cdot \cos(x)$$

$$\Rightarrow f'(x) = f(x) [(\ln 7) \cdot \cos(x)]$$
$$= 7^{\sin x} (\ln 7) \cdot \cos(x).$$

6. Given $f(x) = e^{x^2 \sin x}$.

Then $f'(x) = (x^2 \sin x)' e^{x^2 \sin x}$

$= [2x \sin(x) + x^2 \cos(x)] e^{x^2 \sin x}$

6. (Section 4.2, Problem 16)

7. Given $f(x) = \sin(e^{5x-1})$

Then $f'(x) = (e^{5x-1})' \cdot \cos(e^{5x-1})$

7. (Section 4.2, Problem 20)

$= 5e^{5x-1} \cdot \cos(e^{5x-1})$

8. Given $f(x) = \frac{e^{2x}}{1+e^x}$, Find slope of tangent line at $x=0$.

quotient rule

$f'(x) = \frac{2e^{2x}(1+e^x) - e^x \cdot e^{2x}}{(1+e^x)^2} = \frac{e^{3x}+2e^{2x}}{(1+e^x)^2}$

8. (Section 4.2, Problem 30)

$f'(0) = \frac{e^0+2 \cdot e^0}{(1+e^0)^2} = \frac{1+2}{2^2} = \frac{3}{4}$.

9. Given $f(x) = \ln(x^3-x)$.

$f'(x) = \frac{(x^3-x)'}{x^3-x} = \frac{3x^2-1}{x^3-x}$

9. (Section 4.3, Problem 14)

10. (Section 4.3, Problem 26)

Given $f(x) = x \ln(\sqrt{x})$.

Then $f(x) = 1 \cdot \ln(\sqrt{x}) + x \cdot \dfrac{\frac{+1}{2\sqrt{x}}}{\sqrt{x}}$

product rule

$$= \ln(\sqrt{x}) + x \frac{1}{2x}$$

$$= \ln(\sqrt{x}) + \frac{1}{2}.$$