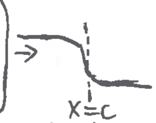
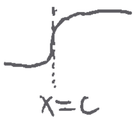
① Vertical Asymptote : $x=c$ if $f(x)\to\pm\infty$ as $x\to c^+$ or $x\to c^-$

② Horizontal Asymptote: $y=L$ if $\lim\limits_{x\to\infty}f(x)=L$ or $\lim\limits_{x\to-\infty}f(x)=L$

③ Vertical tangent @ $x=c$

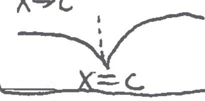
if $\boxed{\lim\limits_{x\to c^+}f'(x)=\infty \text{ and } \lim\limits_{x\to c^-}f'(x)=\infty}$ or $\boxed{\lim\limits_{x\to c^+}f'(x)=-\infty \text{ and } \lim\limits_{x\to c^-}f'(x)=-\infty}$



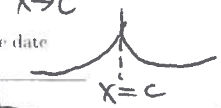


(4) Vertical cusp @ $x=c$,

if $\lim\limits_{x\to c^+}f'(x)=\infty$ and $\lim\limits_{x\to c^-}f'(x)=-\infty$



or $\lim\limits_{x\to c^+}f'(x)=-\infty$ and $\lim\limits_{x\to c^-}f'(x)=\infty$



Homework 8 (10 points)

Due 3/26 in Recitation

Name: Sol PSID:

Instructions:

- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it;
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 3.6, Problem 2) Given $f(x)=\dfrac{x^3}{x+2}$

① Vertical Asymptote: As $x\to-2^+$, $f(x)\to-\infty$.

As $x\to-2^-$, $f(x)\to\infty$

$\Rightarrow x=-2$ is a Vertical Asymptote of f.

② Horizontal: Since $\lim\limits_{x\to\infty}f(x)$ DNE (or ∞), $\lim\limits_{x\to-\infty}f(x)$ DNE (or $-\infty$)

$\Rightarrow f$ has no horizontal asymptote.

2. (Section 3.6, Problem 6) Given $f(x)=\dfrac{\sqrt{x}}{4\sqrt{x}-x}=\dfrac{\sqrt{x}}{\sqrt{x}(4-\sqrt{x})}$

① Vertical Asymptote:

As $x\to16^+$, $f(x)\to-\infty$ and as $x\to16^-$, $f(x)\to\infty$,

then $x=16$ is a vertical Asymptote of f.

③ Horizontal Asymptote:

$\lim\limits_{x\to\infty}f(x)=0 \Rightarrow y=0$ is a horizontal asymptote of f.

3. (Section 3.6, Problem 11) 3. Given $f(x)=\dfrac{3x}{\sqrt{4x^2+1}}$

① Vertical Asymptote: Since $4x^2+1>0$, there is no x such that $f(x)\to\pm\infty$, thus f has no vertical Asymptote.

② Horizontal:

$\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{3x}{\sqrt{4x^2+1}}=\dfrac{3}{2} \Rightarrow y=\dfrac{3}{2}$ is a Horizontal asymptote of f.

same degree

leading coefficient

4. (Section 3.6, Problem 18) Given $f(x)=\dfrac{1}{\sec x-1}$

① Vertical: As $x=2n\pi$, n is an integer, $\sec x-1=0$. then $f(x)\to\infty$. so $x=2n\pi$ are V. asymptotes of f.

② Horizontal: There is no horizontal asymptote of f since $\lim\limits_{x\to\infty}f(x)$ or $\lim\limits_{x\to-\infty}f(x)$ DNE.

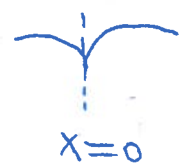
5. (Section 3.6, Problem 20) Given $f(x)=3+x^{\frac{2}{5}}$ @ $x=0$,

Check $f'(x)=\dfrac{2}{5}x^{-\frac{3}{5}}$ as $x\to0^+$ and $x\to0^-$.

$\lim\limits_{x\to0^+}f'(x)=\lim\limits_{x\to0^+}\dfrac{2}{5}x^{-\frac{3}{5}}=\lim\limits_{x\to0^+}\dfrac{2}{5}\cdot\dfrac{1}{x^{\frac{3}{5}}}=\infty$.

and $\lim\limits_{x\to0^-}f'(x)=\lim\limits_{x\to0^-}\dfrac{2}{5}\dfrac{1}{x^{\frac{3}{5}}}=-\infty \Rightarrow$ It is Case (4) (Vertical cusp)

with graph



6. (Section 3.6, Problem 26)

6. Given $f(x) = 4-(2-x)^{\frac{3}{7}}$ at $x=2$

Check $f'(x) = +\frac{3}{7}\frac{1}{(2-x)^{\frac{4}{7}}}$ as $x \to 2^+$ and $x \to 2^-$

$\lim_{x\to 2^+} f'(x) = \lim_{x\to 2^+} \frac{3}{7}\frac{1}{(2-x)^{\frac{4}{7}}} = \infty$ and

$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^-} \frac{3}{7}\frac{1}{(2-x)^{\frac{4}{7}}} = \infty$

$\Rightarrow$ It is Case (3)

Vertical tangent

with graph

$x=2$

7. (Section 3.6, Problem 33)

Given $f(x) = \frac{x+1}{x-2}$, $D(f) = \{x \neq 2\}$

1. Asymptotes $\Rightarrow$ Vertical: $x=2$ ($f(x) \to \pm\infty$ as $x \to 2^{\pm}$)

   Horizontal: $y=1$ (leading coefficient).

2. Intercepts $\Rightarrow$ As $x=0$, $y=-\frac{1}{2}$, As $y=0$, $x=-1$.

   $\Rightarrow$ $(0,-\frac{1}{2})$ and $(-1,0)$.

3. extrema $\Rightarrow f'(x) = \frac{x-2-(x+1)}{(x-2)^2} = \frac{-3}{(x-2)^2} < 0$

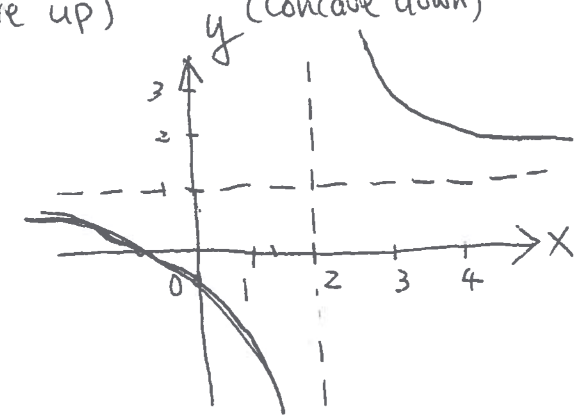
   f is always decreasing except $x=2$. (No extrema)

4. Point of inflection $\Rightarrow f''(x) = \frac{6}{(x-2)^3}$

   $f''(x) > 0$ as $x > 2$. $f''(x) < 0$ as $x < 2$. But NO Point of Inflection.

   (concave up) (concave down)

5. Graphing:



8. (Section 3.6, Problem 34)

8. Given $f(x) = \frac{1}{(x+1)^2}$, $D(f) = \{x \neq -1\}$.

1. Asymptotes $\Rightarrow$ Vertical: $x=-1$ ($f(x) \to \infty$ as $x \to -1^{\pm}$).

   Horizontal: $y=0$ ($\lim_{x\to\pm\infty} f(x) = 0$)

2. Intercepts: as $x=0$, $y=1$.

3. extrema $\Rightarrow f'(x) = \frac{-2}{(x+1)^3}$

   $f'(x) > 0$ as $x < -1$ (increasing) and $f'(x) < 0$ as $x > -1$ (decreasing)

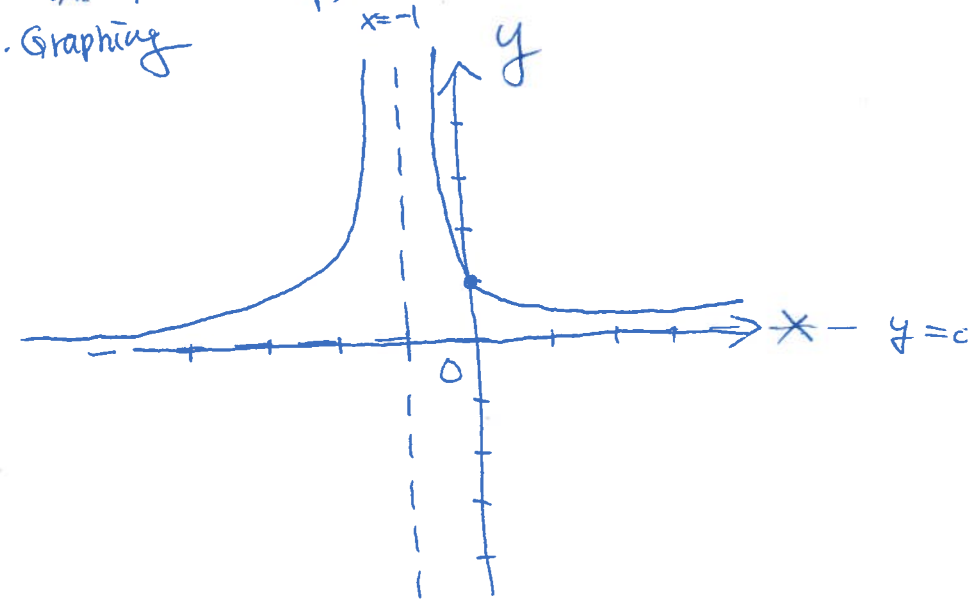
   But No extrema.

4. Point of Inflection $\Rightarrow f''(x) = \frac{6}{(x+1)^4}$.

   $f''(x) > 0$ except $x=-1$.

   (ALWAYS concave up)

5. Graphing



9. Given $f(x)=\frac{x}{1+x^2}$; $D(f)=\{x \in \mathbb{R}\}$

1. Asymptotes $\Rightarrow$ Vertical: None $(1+x^2>0)$.
   Horizontal: $y=0$ $\left(f=\frac{P}{Q}\text{ and } \deg(P)<\deg(Q)\right)$

9. (Section 3.6, Problem 35)

2. Intercepts: As $x=0$, $y=0$. $\Rightarrow$ through $(0,0)$

3. extrema $\Rightarrow f'(x)=\frac{1+x^2-2x^2}{(1+x^2)^2}=\frac{1-x^2}{(1+x^2)^2}$

$\Rightarrow f'(x)=0 \Leftrightarrow 1-x^2=0 \Rightarrow (1-x)(1+x)=0$, $x=\pm 1$

$f'(x)$: $- - - -$ $\underset{-1}{|}$ $+ + + + + +$ $\underset{1}{|}$ $- - - -$

local min $f(-1)=-\frac{1}{2}$ local max $f(1)=\frac{1}{2}$

4. Point of inflection $\Rightarrow f''(x)=\frac{-2x(1+x^2)^2-4x(1+x^2)(1-x^2)}{(1+x^2)^4}$

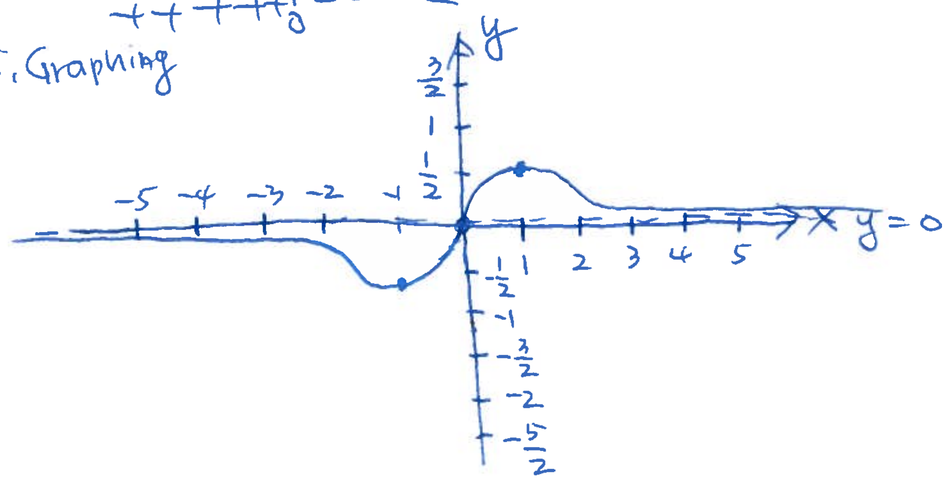
$=\frac{-2x(1+x^2)[(1+x^2)+1-x^2]}{(1+x^2)^4}$

$=\frac{-2x\cdot 2}{(1+x^2)^3}$

$f''(x)=0 \Rightarrow x=0$ is a point of inflection $(0,0)$

concave up $+ + + + + +$ $\underset{0}{|}$ $- - - -$ concave down

5. Graphing



10. Given $f(x)=\frac{x-2}{x^2-5x+6}=\frac{(x-2)}{(x-2)(x-3)}=\frac{1}{(x-3)}$

$D(f)=\{x\neq 2, x\neq 3\}$ And $x=2$ is a Removable discontinuity

1. Asymptotes $\Rightarrow$ Vertical: $x=3$ $(f\to\pm\infty$ as $x\to 3^{\pm})$
   Horizontal: $y=0$ $\left(f=\frac{P}{Q}\text{ and } \deg(P)<\deg(Q)\right)$

10. (Section 3.6 Problem 36)

2. Intercepts As $x=0$, $y=-\frac{1}{3}$. $\Rightarrow (0,-\frac{1}{3})$

3. extrema $\Rightarrow f'(x)=\frac{-1}{(x-3)^2}<0$.

$\Rightarrow f$ is always decreasing except $x=3$ & 2.

4. point of Inflection $\Rightarrow$

$f''(x)=\frac{2}{(x-3)^3}$

$f''(x)>0$ as $x>3$, concave up. $f''(x)<0$ as $x<3$ concave down.

But 3 is NOT A POINT OF INFLECTION.

5. Graphing: