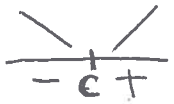
Critical point c: We can find extreme value at critical points c
($f'(c)=0$ or DNE)

First derivative test:
① $f'(x)$: $-\ c\ +$ local min.



② $f'(x)$ $+\ c\ -$ local max

③ $f'(x)$ NOT change: None of above

F.D.T.

Second derivative test: ① $f''(c)<0$ (concave down) ⇒ local max.
② $f''(c)>0$ (concave up) ⇒ local min.
③ $f''(c)=0$: go back to first derivative test

Math 1431, Section 17699

Homework 7 (10 points)

Due 3/12 in Recitation

Name: ______________________ PSID: Sol

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

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1. (Section 3.4, Problem 2) Given $f(x)=x+\frac{1}{x}$. $f'(x)=1-\frac{1}{x^2}$.

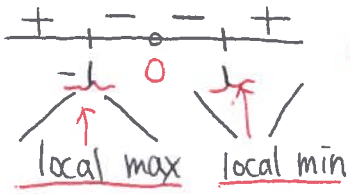
$D(f)=\{x\neq 0\}$ (Domain of f).

(a). Critical numbers: △ $f'(x)=0 \Rightarrow 1-\frac{1}{x^2}=0 \Rightarrow \boxed{x=\pm 1.}$

△ $f'(x)$ DNE. $x\cancel{=}0$ (NOT IN DOMAIN)

(b) Extreme Value: $f(1)=1+\frac{1}{1}=2$, $f(-1)=1-1=0$.

(c) First Derivative Test ⇒ Number line of $f'(x)$



2. (Section 3.4, Problem 5) Given $f(x)=\frac{2}{x(x+1)}$

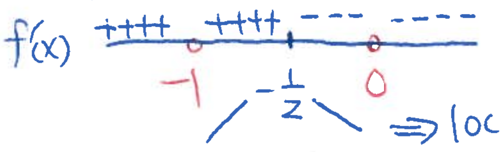
$D(f)=\{x\neq 0, x\neq -1\}$, $f'(x)=\frac{-4x-2}{x^2(x+1)^2}$

(a) Critical numbers: △ $f'(x)=0 \Rightarrow -4x-2=0 \Rightarrow x=-\frac{1}{2}$

△ $f'(x)$ DNE, ⇒ $x=\cancel{0,}\ -1$ (NOT IN DOMAIN)

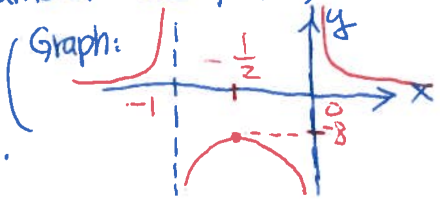
(b) Extreme Value: $f(-\frac{1}{2})=\frac{2}{(-\frac{1}{2})\cdot(\frac{1}{2})}=-8$.

(c) First Derivative Test ⇒ Number line of $f'(x)$:



⇒ local max.

(Graph:



)

3. (Section 3.4, Problem 6) Given $f(x)=x^3(1-x)^2$, $D(f)=\mathbb{R}$ or $(-\infty,\infty)$

$f'(x)=3x^2(1-x)^2-2x^3(1-x)=x^2(1-x)(3-5x)$

(a) Critical Numbers: △ $f'(x)=0 \Rightarrow \boxed{x=0,1,\frac{3}{5}}$

△ $f'(x)$ DNE: NONE.

(b) Extreme Value: $f(0)=0$, $f(1)=0$, $f(\frac{3}{5})=(\frac{3}{5})^3(\frac{2}{5})^2$.

(c) First Derivative Test: Number line of $f'(x)$ ⇒

4. (Section 3.4, Problem 12) Given $f(x)=(x-1)^{\frac{2}{3}}$, $D(f)=\mathbb{R}$ or $(-\infty,\infty)$.

$f'(x)=\frac{2}{3}\frac{1}{(x-1)^{\frac{1}{3}}}$

(a) Critical points: $f'(x)=0$: NONE.
$f'(x)$ DNE ⇒ $x=1$.

(b) Extreme Value: $f(1)=0$ local min

(c) F.D.T.

Number line of $f'(x)$

5. (Section 3.4, Problem 15)

$$\text{Given } f(x)=\begin{cases} 2-2x-x^2 & -2\leq x\leq 0 \\ |x-2| & 0<x<3 \\ \frac{1}{3}(x-2)^3 & 3\leq x\leq 4 \end{cases}$$

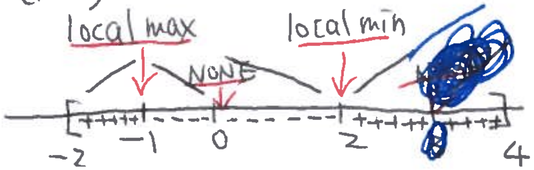
$D(f)=(-2,4)$.

$$f'(x)=\begin{cases} -2-2x & -2\leq x<0 \\ \text{DNE} & x=0 \\ -1 & 0<x<2 \\ \text{DNE} & x=2 \\ 1 & 2<x<3 \\ \cancel{\text{DNE}}\ 1 & x=3 \\ (x-2)^2 & 3<x\leq 4 \end{cases}$$

(a) Critical points:
△ $f'(x)=0$; $x=-1$, $\cancel{x=2}$
△ $f'(x)$ DNE: $x=0,2,$ [illegible]

(b) $f(-1)=3$, $f(0)=2$
$f(2)=0$ [illegible]

(c)



6. Given $f(x)=\sin(x)\cos(x)$ on $(0,\pi)$.

$D(f)=(0,\pi)$. $f'(x)=\cos(x)\cos(x)+\sin(x)\cdot(-\sin(x))$

$=\cos^2(x)-\sin^2(x)-\cos^2(x)+\cos^2(x)$

$=2\cos^2(x)-1$

6. (Section 3.4, Problem 24)

(a) Critical points: △ $f'(x)=0 \Rightarrow 2\cos^2(x)=1 \Rightarrow \cos(x)=\pm\frac{\sqrt{2}}{2}$

$\Rightarrow x=\frac{\pi}{4},\frac{3\pi}{4}$.

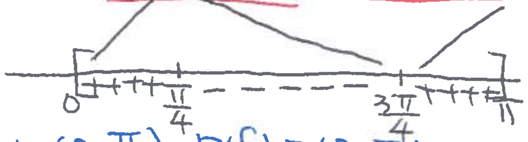
△ $f'(x)$ DNE: NONE.

(b) Extreme Value: $f(\frac{\pi}{4})=\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}=\frac{1}{2}$; $f(\frac{3\pi}{4})=-\frac{1}{2}$

(c) F.D.T: Number line of $f'(x)$:

local max

local min



7. (Section 3.4. Problem 30)

Given $f(x)=\frac{\sin(x)}{1+\cos^2(x)}$ on $(0,\pi)$. $D(f)=(0,\pi)$.

$f'(x)=\frac{\cos(x)[2+\sin^2(x)]}{[1+\cos^2(x)]^2}$ (See #10, HW6).

(a) $f'(x)=0 \Leftrightarrow \cos(x)=0 \Rightarrow x=\frac{\pi}{2}$ → Number line of $f(x)$

(b) Extreme Value

(c) F.D.T

$f(\frac{\pi}{2})=\frac{1}{1+0}=1$.

local max

8. (Section 3.5, Problem 4)

Given $f(x)=2x^2-5x+2$. $\Rightarrow f'(x)=4x-5$. $f''(x)=4$.

Since $f''(x)>0$ for all x in ℝ.

Then f is always concave down or

f is concave down on $(-\infty,\infty)$

and there is NO Inflection point.

9. Given $f(x)=\frac{x}{x^2-1}$; $D(f)=\{x\neq\pm1\}$

$=\frac{x}{(x+1)(x-1)}$

$f'(x)=\frac{-x^2-1}{(x^2-1)^2}$, $f''(x)=\frac{\cancel{(x^2-1)}(2x^3+6x)}{(x^2-1)^{\cancel{4}3}}$

9. (Section 3.5, Problem 7)

$f''(x)=0 \Leftrightarrow 2x^3+6x=0 \Rightarrow 2x(x^2+3)=0$

$\Rightarrow x=0$.

$f''(x)$ DNE $\Rightarrow x=\cancel{\pm}1$, (NOT IN DOMAIN)

Number line of $f''(x)$

(c) $x=0$ is a pt of inflection

(b) concave down ($f''<0$) : $(-\infty,-1)\cup(0,1)$.

(a) concave up ($f''>0$) : $(-1,0)\cup(1,\infty)$.

10. (Section 3.5 Problem 14)

Given $f(x)=x\sqrt{4-x^2}$

$D(f)=\{4-x^2\geq0\}$

$=\{-2\leq x\leq 2\}$

$f'(x)=\sqrt{4-x^2}+\frac{1}{2}\frac{-2x^2}{\sqrt{4-x^2}}$

$f''(x)=\frac{-2x}{2}\frac{1}{\sqrt{4-x^2}}+\frac{-4x}{2}\frac{1}{\sqrt{4-x^2}}+\frac{-2x^2}{2}\cdot-\frac{1}{2}\frac{-2x}{(\sqrt{4-x^2})^3}$

$=\frac{-3x(4-x^2)-x^3}{(\sqrt{4-x^2})^3}=\frac{\cancel{-}4x^3-12x}{(\sqrt{4-x^2})^3}=\frac{2x^3-12x}{(\sqrt{4-x^2})^3}$

$f''(x)$ DNE $\Rightarrow$ $x=\pm2$

$f''(x)=0 \Rightarrow \cancel{-4x^3-12x}=0$ $\Rightarrow 2x^3-12x=0$

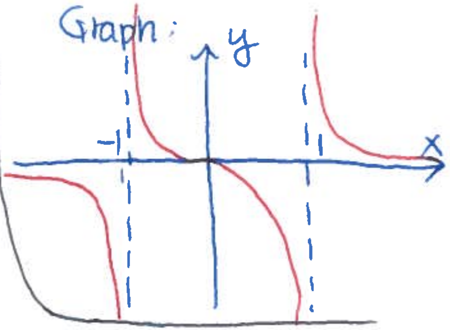
$\Rightarrow \cancel{-4x(x^2+3)}=0 \Rightarrow \cancel{x=0}$

$2x(x+\sqrt{6})(x-\sqrt{6})=$

$\Rightarrow x=0$ or $\sqrt{6}$ or $-\sqrt{}$

NOT IN DOMA

Graph:



Number line of $f''(x)$

Concave up: $[-2,0)$.

Concave down: $(0,2)$.

Inflection: 0