(*) $\cos(2x) = \cos(x+x)$

$$= \cos(x)\cdot\cos(x) - \sin(x)\cdot\sin(x)$$

$$= \cos^2(x) - \sin^2(x) = \cos^2(x) - \sin^2(x) - \cos^2(x) + \cos^2(x)$$

$$= 2\cos^2(x) - 1$$

Math 1431, Section 17699

Homework 6 (10 points)

Due 3/5 in Recitation

Name: Seol PSID:

Instructions:

- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it;
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 3.2, Problem 4) Given $f(x) = -\frac{x^3}{3} + 4x + 1$. Find x s.t. $f'(x) = 0$

$$f'(x) = -x^2 + 4 = 0 \Rightarrow x^2 - 4 = 0$$

$$\Rightarrow (x+2)(x-2) = 0 \Rightarrow \boxed{x = 2 \text{ or } -2}$$

2. For Rolles Theorem, first check $f(a) = f(b)$ then find $c \in (a,b)$ s.t. $f'(c) = 0$

2. (Section 3.2, Problem 18)

Given $f(x) = \cos 2x$, on $[-\frac{\pi}{12}, \frac{\pi}{6}]$ ($a = -\frac{\pi}{12}$, $b = \frac{\pi}{6}$)

$$f(a) = f(-\tfrac{\pi}{12}) = \cos(-\tfrac{\pi}{6}) = \frac{\sqrt{3}}{2}$$

$$f(b) = f(\tfrac{\pi}{6}) = \cos(2\cdot\tfrac{\pi}{6}) = \cos(\tfrac{\pi}{3}) = \frac{1}{2}$$

$f(a) \neq f(b)$. We can't apply Rolles thm on it.

3. For Mean value theorem (M.V.T)
If f is continuous on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$.

3. (Section 3.2, Problem 22)

Given $f(x) = 3\sqrt{25-x^2}$ on $[0, 5]$ ($a = 0$, $b = 5$) which is satisfied the assumptions of MVT. Then

$$f'(x) = \frac{3}{2}\frac{-2x}{\sqrt{25-x^2}}, \quad f(a) = f(0) = 3\cdot 5 = 15, \quad f(5) = 0$$

$$\frac{3}{2}\frac{-2c}{\sqrt{25-c^2}} = f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{0-15}{5-0} = -3$$

doing cross product, we have $c = \sqrt{25-c^2}$

$$\Rightarrow c^2 = 25 - c^2 \Rightarrow 2c^2 = 25$$

$$\boxed{c = \frac{5}{2}\sqrt{2}}$$

4. (Section 3.2, Problem 30)

4. Given $f(x) = 2\sin(x) + \sin(2x)$ on $[0, \pi]$

$$f'(x) \overset{(*)}{=} 2\cos(x) + 2\cos(2x) = 2\cos(x) + 4\cos^2(x) - 2, \quad f(a) = f(0) = 0, \quad f(b) = f(\pi) = 0$$

$$2\cos(c) + 4\cos^2(c) - 2 = f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{0}{\pi} = 0$$

$$2\cos^2(c) + \cos(c) - 1 = 0 \Rightarrow (\cos(c)+1)(2\cos(c)-1) = 0$$

$$\Rightarrow \cos(c) = -1 \text{ or } \frac{1}{2}$$

$$c = \cancel{\pi} \text{ or } \boxed{\frac{\pi}{3}}$$

5. (Section 3.3, Problem 2)

5. Given $f(x) = x^2 + 8x + 10$

$$f'(x) = 2x + 8 \Rightarrow 2(x+4)$$

Increasing interval $\Rightarrow f'(x) > 0 \Rightarrow 2(x+4) > 0 \Rightarrow x > -4$ or $x \in (-4, \infty)$

Decreasing interval $\Rightarrow f'(x) < 0 \Rightarrow 2(x+4) < 0 \Rightarrow x < -4$ or $x \in (-\infty, -4)$

6. Given $f(x)=x^3-6x^2+15$.

$f'(x)=3x^2-12x=3x(x-4)$.

Increasing interval $\Rightarrow f'(x)>0 \Rightarrow x\in(-\infty,0)\cup(4,\infty)$

6. (Section 3.3, Problem 6)

Decreasing interval $\Rightarrow f'(x)<0 \Rightarrow x\in(0,4)$

Given $f(x)=|x+3|-1=\begin{cases} x+3-1, & x>3; \\ -1, & x=3; \\ -x-3-1, & x<3. \end{cases}$

7. (Section 3.3, Problem 16)

$=\begin{cases} x+2, & x>3; \\ -1, & x=3; \\ -x-4, & x<3. \end{cases}$

$\Rightarrow f'(x)=\begin{cases} 1, & x>3; \\ DNE, & x=3; \\ -1, & x<3, \end{cases}$

Increasing interval $\Rightarrow f'(x)>0, \quad x>3,$

Decreasing Interval $\Rightarrow f'(x)<0, \quad x<3.$

8. (Section 3.3, Problem 18)

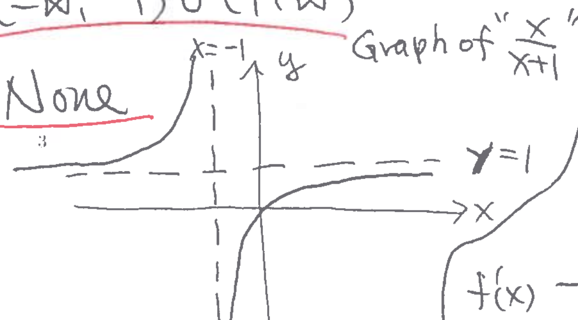
$f(x)=\frac{x}{x+1}$: Domain of $f=\{x\in\mathbb{R} \mid x\neq-1\}$

quotient rule

$f'(x)=\frac{(x+1)-x}{(x+1)^2}=\frac{1}{(x+1)^2}>0$ (ALWAYS more than 0)

Increasing interval : $(-\infty,-1)\cup(1,\infty)$

Decreasing interval : None



9. Given $f(x)=\sqrt{3}\sin(x)+\cos(x)$ on $(0,2\pi)$.

$f'(x)=\sqrt{3}\cos(x)-\sin(x)$

As $f'(x)=0$, we have $\sqrt{3}\cos(x)-\sin(x)=0$

$\Rightarrow \sqrt{3}\cos(x)=\sin(x) \Rightarrow \tan(x)=\sqrt{3}$.

9. (Section 3.3, Problem 28)

$\Rightarrow x=\frac{\pi}{3}$ or $\frac{4\pi}{3}$

$\Rightarrow$ Increasing interval, $x\in(0,\frac{\pi}{3})\cup(\frac{4\pi}{3},2\pi)$.

Decreasing interval $x\in(\frac{\pi}{3},\frac{4\pi}{3})$.

10. (Section 3.3 Problem 30)

Given $f(x)=\frac{\sin(x)}{1+\cos^2(x)}$ on $(0,2\pi)$.

(since $1+\cos^2(x)\neq0$. Domain of f is $\mathbb{R}$)

$f'(x)=\frac{\cos(x)[1+\cos^2(x)]-[-2\sin(x)\cos(x)]\sin(x)}{[1+\cos^2(x)]^2}$

$=\frac{\cos(x)+\cos^3(x)+2\sin^2(x)\cos(x)}{[1+\cos^2(x)]^2}$

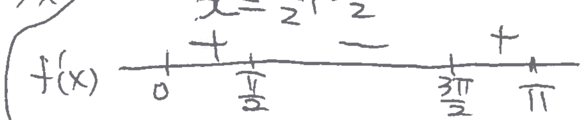
$=\frac{\cos(x)[1+\cos^2(x)+2\sin^2(x)]}{[1+\cos^2(x)]^2}$.

As $f'(x)=0 \Leftrightarrow \cos(x)[1+\cos^2(x)+2\sin^2(x)]=0$

$\Leftrightarrow \cos(x)[1+\underbrace{\cos^2(x)+\sin^2(x)}_{1}+\sin^2(x)]=0$

$\Leftrightarrow \cos(x)\underbrace{[2+\sin^2(x)]}_{\text{always}>0}=0 \Leftrightarrow \cos(x)=0$

$x=\frac{\pi}{2},\frac{3\pi}{2}$



Increasing interval $(0,\frac{\pi}{2})\cup(\frac{3\pi}{2},\pi)$

Decreasing interval $(\frac{\pi}{2},\frac{3\pi}{2})$