Math 1431, Section 17699

Homework 5 (10 points)

Due 2/26 in Recitation

Name: ______________________ PSID: ____________________

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

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1. (Section 2.4, Problem 8) Given $x^3y^3 - y = x$. Find $\frac{dy}{dx}$.

do "$\frac{d}{dx}$" on both sides. $\frac{d}{dx}(x^3y^3 - y) = \frac{d}{dx}(x)$

$\Rightarrow \frac{d}{dx}(x^3y^3) - \frac{d}{dx}(y) = 1 \Rightarrow 3x^2y + x^3 \cdot 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1$

product rule $\Rightarrow (3x^3y^2 - 1)\frac{dy}{dx} = 1 - 3x^2y$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y}{3x^3y^2 - 1}}$$

2. (Section 2.4, Problem 10)

Given $x^3 - 3x^2y + 2xy^2 = 12$, Find $\frac{dy}{dx}$.

$\frac{d}{dx}(x^3 - 3x^2y + 2xy^2) = \frac{d}{dx}(12) \Rightarrow \underbrace{\frac{d}{dx}(x^3)}_{①} - \underbrace{\frac{d}{dx}(3x^2y)}_{②} + \underbrace{\frac{d}{dx}(2xy^2)}_{③} = 0$

① ② product rule ②

$\Rightarrow 3x^2 - 6xy - 3x^2\frac{dy}{dx} + 2y^2 + 2x \cdot 2y\frac{dy}{dx} = 0$

$\Rightarrow (4xy - 3x^2)\frac{dy}{dx} = 6xy - 3x^2 - 2y^2.$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{6xy - 3x^2 - 2y^2}{(4xy - 3x^2)}}$$

3. (Section 2.4, Problem 12) Given $\sin x + 2\cos 2y = 1$. Find $\frac{dy}{dx}$.

$\frac{d}{dx}(\sin(x) + 2\cos(2y)) = \frac{d}{dx}(1) = 0 \Rightarrow \frac{d}{dx}(\sin(x)) + 2\frac{d}{dx}(\cos(2y)) =$

$\Rightarrow \cos(x) \cdot \frac{dx}{dx}^{1} + 2 \cdot (-\sin(2y)) \cdot 2 \cdot \frac{dy}{dx} = 0$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\cos(x)}{4\sin(2y)}}$$

4. (Section 2.4, Problem 14) Given $\sin x = x(1 + \tan(y))$. Find $\frac{dy}{dx}$.

$\frac{d}{dx}(\sin(x)) = \frac{d}{dx}[x(1 + \tan(y))]$ product rule

$\Rightarrow \cos(x) = (1 + \tan(y)) + x \cdot \frac{d}{dx}(\tan(y))$

$\Rightarrow \cos(x) = (1 + \tan y) + x \cdot (\sec^2(y)) \cdot \frac{dy}{dx}$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\cos(x) - 1 - \tan y}{x\sec^2(y)}}$$

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5. (Section 2.4, Problem 30) Given $(x^2+4)y = 8$ and point $(2,1)$. Find tangent line @ $(2,1)$

slope: ~~at (2,1)~~ $\frac{dy}{dx} \Rightarrow$ do "$\frac{d}{dx}$": $\frac{d}{dx}[(x^2+4)y] = \frac{d}{dx}(8)$

$\Rightarrow \left[\frac{d}{dx}(x^2+4)\right]y + (x^2+4)\frac{dy}{dx} = 0$

$\Rightarrow 2xy + (x^2+4)\frac{dy}{dx} = 0$

slope at $(2,1) \Rightarrow 2 \cdot 2 \cdot 1 + (2^2+4)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

tangent line: $\boxed{(y-1) = -\frac{1}{2}(x-2)}$.

6. Given graph $4x^2+y^2-8x+4y+4=0$

① Find horizontal ~~tangent~~ line $\Rightarrow \frac{dy}{dx}=0$

② Find vertical ~~tangent~~ line $\Rightarrow \frac{dx}{dy}=0$

6. (Section 2.4, Problem 48)

Find $\frac{dy}{dx} \Rightarrow \frac{d}{dx}(4x^2+y^2-8x+4y+4)=0 \Rightarrow 8x+2y\frac{dy}{dx}-8+4\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{8-8x}{2y+4}$.

① $\frac{dy}{dx}=0 \Rightarrow x=1,\ y^2+4y=0 \Rightarrow y=0$ or -4

put it back to the Given Graph!

points for horizontal tangent: $(1,0)$ or $(1,-4)$

② $\frac{dx}{dy}=0 \Rightarrow y=-2 \quad 4x^2-8x=0 \Rightarrow x=0$ or 2

put it back to the Graph!

points for vertical tangent: $(0,-2)$, $(2,-2)$

7. (Section 3.1, Problem 8)

Given position of a object $x(t)=\frac{1}{4}t^4-t^3+t^2$, $t\geq 0$

Find the time interval s.t. object moves right. $\Rightarrow$ Velocity >0

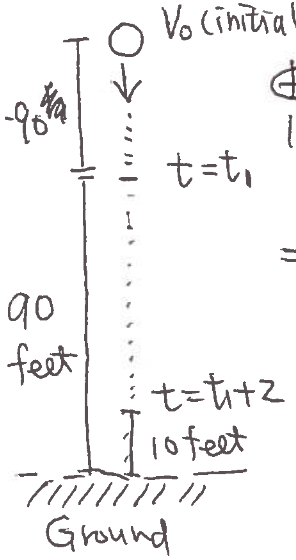
$\Rightarrow x'(t)>0 \Rightarrow x'(t)=t^3-3t^2+2t \;=t(t^2-3t+2)$

$=t(t-1)(t-2)>0 \Rightarrow \boxed{t\in(0,1)\cup(2,\infty)}$

$-\ \ +\ \ -\ \ +$

$0 \quad 1 \quad 2$

8. (Section 3.1, Problem 16)



V_0 (initial speed) $=4$, t (time) $=0$

~~let h be the length of path from t=0 to t₁~~

let h be the height of the stone at the beginning.

gravity of earth is $g=32.2\ ft/s^2$

$\Rightarrow \begin{cases} h-90=4\cdot t_1+\frac{1}{2}gt_1^2 & \Rightarrow h=4t_1+\frac{g}{2}t_1^2+90 \quad -(1) \\ h-10=4\cdot(t_1+2)+\frac{1}{2}g(t_1+2)^2 & \Rightarrow h=4(t_1+2)+\frac{g}{2}(t_1+2)^2+10 \quad -(2) \end{cases}$

$\Rightarrow 4t_1+\frac{g}{2}t_1^2+90=4(t_1+2)+\frac{g}{2}(t_1+2)^2+10$

$\Rightarrow 90=8+2gt_1+2g+10 \Rightarrow t_1=\frac{72-2g}{2g}$

put it back to (1): $g=32.2$

$h=4\cdot\left(\frac{72-2g}{2g}\right)+\frac{g}{2}\cdot\left(\frac{72-2g}{2g}\right)^2+90=\boxed{90.6962}$

9. Given $y^2=4(x+6)$ and $\left.\frac{dy}{dt}\right|_{(10,8)}=3\frac{\text{units}}{\text{second}}$

Find $\left.\frac{dx}{dt}\right|_{(10,8)}$

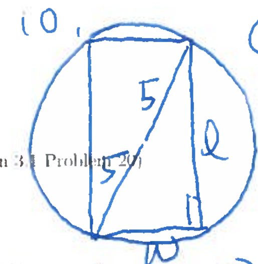
9. (Section 3.1, Problem 18)

do "$\frac{d}{dt}$" on both sides, $2y\cdot\frac{dy}{dt}=4\cdot\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt}=\frac{2y}{4}\frac{dy}{dt} \Rightarrow \left.\frac{dx}{dt}\right|_{(10,8)}=\frac{2\cdot 8}{4}\cdot 3$

$\boxed{=12}$

10. (Section 3.1, Problem 20)



Given $\frac{dl}{dt}=-2$

Find $\left.\frac{dA}{dt}\right|_{l=6}$.

We have $l^2+w^2=10^2$ and $A=lw$.

$\Rightarrow w=\sqrt{100-l^2} \Rightarrow A=l\cdot\sqrt{100-l^2}$

$\frac{dA}{dt}=\frac{dl}{dt}\cdot\sqrt{100-l^2}+l\cdot\frac{1}{2}(100-l^2)^{-\frac{1}{2}}\cdot(-2l)\cdot\frac{dl}{dt}$

$\left.\frac{dA}{dt}\right|_{l=6}=-2\cdot\sqrt{100-36}+6\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{100-36}}\cdot(-2\cdot 6)\cdot(-2)$

$=-16+9=\boxed{-7}$

10. Another Way to deal with this:

Given $\frac{dl}{dt} = -2$. Find $\left.\frac{dA}{dt}\right|_{l=6} \underset{\substack{\uparrow \\ A=lw}}{=} \left.\frac{d(lw)}{dt}\right|_{l=6} = \left.\frac{dl}{dt}\right|_{l=6} w + \left.\frac{dw}{dt}\right|_{l=6} \cdot l$ —— (*)

(need $\left.\frac{dw}{dt}\right|_{l=6}$ and w as $l=6$)

Since $l^2+w^2=100$, So, as $l=6$, $w=8$

Furthermore, do "$\frac{d}{dt}$" to "$l^2+w^2=100$", we obtain. $2l\frac{dl}{dt}+2w\frac{dw}{dt}=0$.

as $\underline{l=6, w=8}$, $\Longrightarrow$ $2\cdot 6\cdot(-2)+2\cdot 8\cdot \frac{dw}{dt}=0 \Rightarrow \frac{dw}{dt}=\frac{24}{16}=\frac{3}{2}$,

put those informations back to (*). we get

$$\left.\frac{dA}{dt}\right|_{\substack{l=6\\w=8}} = \left.\frac{dl}{dt}\right|_{\substack{l=6\\w=8}} \cdot 8 + \left.\frac{dw}{dt}\right|_{\substack{l=6\\w=8}} \cdot 6$$

$$= (-2)\cdot 8 + \frac{3}{2}\cdot 6 = -16+9 = -7$$