Math 1431, Section 17699

Homework 4 (10 points)

Due 2/19 in Recitation

Name: ________ PSID: Sol

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 2.2, Problem 6)

$f(x) = \frac{4}{x^2} - 10x^3 = 4x^{-2} - 10x^3$

$\boxed{f'(x) = -8x^{-3} - 30x^2}$

2. (Section 2.2, Problem 11)

Given $f(x) = 2\sin x$ and point $(\frac{\pi}{3}, f(\frac{\pi}{3}))$
$= (\frac{\pi}{3}, \sqrt{3})$

Slope of ~~tangent~~ line: $f'(x) = 2\cos x$

at $x = \frac{\pi}{3}$: $f'(\frac{\pi}{3}) = 2\cos\frac{\pi}{3} = 2\cdot\frac{1}{2} = 1$.

equation of line: $\boxed{(y - \sqrt{3}) = 1\cdot(x - \frac{\pi}{3})}$

3. (Section 2.2, Problem 12)

3. Given $f(x) = 6\sqrt{x} = 6\cdot x^{\frac{1}{2}}$ and point $(4, f(4)) = (4, 12)$

Slope of ~~tangent~~ line: $f'(x) = 6\cdot\frac{1}{2}x^{-\frac{1}{2}} = 3x^{-\frac{1}{2}} = 3\frac{1}{\sqrt{x}}$

at $x = 4$: $f'(4) = 3\cdot\frac{1}{\sqrt{4}} = 3\cdot\frac{1}{2} = \frac{3}{2}$

equation of line:

$\boxed{(y - 12) = \frac{3}{2}(x - 4).}$

4. (Section 2.2, Problem 15)

Given $f(x) = x^3 + 4x^2 + 3$ and point $(1, f(1)) = (1, 8)$

$f'(x) = 3x^2 + 2\cdot 4x + 0$

m_t = Slope of ~~tangent~~ line @ $x = 1$: $f'(1) = 3\cdot 1^2 + 8\cdot 1^2 = 11$.

m_n = Slope of normal line: $\frac{-1}{m_t} = -\frac{1}{11}$.

normal line: $\boxed{(y - 8) = -\frac{1}{11}(x - 1)}$

5. (Section 2.2, Problem 18)

Given $f(x) = x^4 - 8x^2 + 13$ ~~and point~~

Find x such that the ~~tangent~~ line is horizontal

$\Longleftrightarrow$ Find x such that $f'(x) = 0$

$0 = f'(x) = 4x^3 - 16x + 0$

$\Rightarrow 4x(x^2 - 4) = 0$

$\Rightarrow 4x(x+2)(x-2) = 0$

$\Rightarrow \boxed{x = 0 \text{ or } -2 \text{ or } 2.}$

6. Given $f(x)=\sqrt[3]{x} = x^{\frac{1}{3}}$, $g(x)=x^2+3$.

Assume $h=f\cdot g$

Then $h'(x) \overset{\text{product rule}}{=} f'(x)g(x)+g'(x)f(x)$

6. (Section 2.3, Problem 4)

$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$

$g'(x)=2x$

$= \boxed{\frac{1}{3}x^{-\frac{2}{3}}\cdot(x^2+3)+(2x)\sqrt[3]{x}}$

7. Given $f(x)=1-\sin(x)$, $g(x)=2\cos(x)$.

7. (Section 2.3, Problem 10)

and $h(x)=\frac{f(x)}{g(x)}$, then $h'(x)=\frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$.

$f'(x)=-\cos(x)$

$g'(x)=-2\sin(x)$

$$=\frac{-\cos(x)\cdot 2\cos(x)-(-2\sin(x))(1-\sin(x))}{[2\cos(x)]^2}$$

$$=\frac{-2\cos^2(x)-2\sin^2(x)+2\sin(x)}{4\cos^2(x)}$$

$$=\frac{-2+2\sin(x)}{4\cos^2(x)}=\boxed{\frac{-1+\sin(x)}{2\cos^2(x)}}$$

8. (Section 2.3, Problem 14)

Given $f(x)=x^4\cot(4x)$.

$f'(x) \overset{\text{product rule}}{=} (x^4)'\cot(4x)+x^4\cdot[\cot(4x)]'$

$\overset{\text{chain rule}}{=} 4x^3\cot(4x)+x^4\cdot[-\csc(4x)\cdot\cot(4x)]\cdot(4x)'$

$=\boxed{4x^3\cot(4x)-4x^4\csc(4x)\cot(4x)}$

9. Given $f(x)=\sqrt{3+\sin(5x)}=(3+\sin(5x))^{\frac{1}{2}}$

$f'(x) \overset{\text{chain rule}}{=} \frac{1}{2}(3+\sin(5x))^{-\frac{1}{2}}\cdot[3+\sin(5x)]'$

9. (Section 2.3, Problem 16)

$\overset{\text{chain rule}}{=} \frac{1}{2}(3+\sin(5x))^{-\frac{1}{2}}\cdot\cos(5x)\cdot[5x]'$

$$=\boxed{\frac{5}{2}\cos(5x)(3+\sin(5x))^{-\frac{1}{2}}}$$

10. (Section 2.3 Problem 32)

Given $f(x)=\frac{\sin(6x)}{1+\cos(3x)}$ and $c=0$.

Find $f'(0)$.

first. $f'(x) \overset{\text{quotient rule}}{=} \frac{[\sin(6x)]'(1+\cos(3x))-[1+\cos(3x)]'\sin(6x)}{[1+\cos(3x)]^2}$

$\overset{\text{chain rule}}{=} \frac{6\cos(6x)(1+\cos(3x))-[-3\sin(3x)]\sin(6x)}{[1+\cos(3x)]^2}$

$$f'(0)=\frac{6\cos(0)(1+\cos(0))+3\sin(0)\cdot\sin(0)}{[1+\cos(0)]^2}$$

$$=\frac{(6\cdot1)(1+1)+3\cdot0\cdot0}{[1+1]^2}=\frac{6\cdot2}{2^2}=\frac{12}{4}=3$$