$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

Math 1431, Section 17699

Homework 3 (10 points)

Due 2/12 in Recitation

Name: ____________ PSID: Sol.

Instructions:

- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 1.6, Problem 2)

$$\lim_{x\to 0}\frac{\sin(2x)}{3x} = \lim_{x\to 0}\frac{\sin(2x)}{3x}\cdot\frac{2x}{2x} = \lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot\frac{2x}{3x}$$

$$= 1\cdot\frac{2}{3} = \frac{2}{3}$$

2. (Section 1.6, Problem 6)

$$\lim_{x\to 0}\frac{2x}{\tan(3x)} = \lim_{x\to 0}\frac{2x}{\frac{\sin(3x)}{\cos(3x)}} = \lim_{x\to 0}\frac{2x}{\sin(3x)}\cdot\cos(3x)$$

$$= \lim_{x\to 0}\frac{2x}{\sin(3x)}\cdot\cos(3x)\cdot\frac{3}{3} = \lim_{x\to 0}\frac{3x}{\sin(3x)}\cdot\cos(3x)\cdot\frac{2}{3}$$

$$= 1\cdot 1\cdot\frac{2}{3} = \frac{2}{3}$$

$1-\sec^2(7x) = -\tan^2(7x)$

3. (Section 1.6, Problem 10)

$$\lim_{x\to 0}\frac{1-\sec^2(7x)}{(3x)^2} = \lim_{x\to 0}\frac{-\tan^2(7x)}{(3x)^2}$$

$$= \lim_{x\to 0} -\left(\frac{\tan(7x)}{3x}\right)^2 = -\left(\lim_{x\to 0}\frac{\tan(7x)}{3x}\right)^2 = -\left(\frac{7}{3}\right)^2 = -\frac{49}{9}$$

$$\lim_{x\to 0}\frac{\tan(7x)}{3x} = \lim_{x\to 0}\frac{\sin(7x)}{\cos(7x)}\cdot\frac{1}{3x}\cdot\frac{7x}{7x} = \lim_{x\to 0}\frac{\sin(7x)}{7x}\cdot\frac{1}{\cos(7x)}\cdot\frac{7x}{3x} = \frac{7}{3}$$

4. (Section 1.6, Problem 16)

$$\lim_{x\to 0}\frac{x+2}{\cot x} = \lim_{x\to 0}(x+2)\cdot\frac{\sin x}{\cos x} = \lim_{x\to 0}\frac{(x+2)\sin x}{\cos x}\cdot\frac{x}{x}$$

$$= \lim_{x\to 0}\frac{x(x+2)}{\cos x}\frac{\sin x}{x} = \lim_{x\to 0}\left(\frac{x(x+2)}{\cos x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$

$$= 0\cdot 1 = 0$$

5. (Section 1.6, Problem 19)

Given $f(x)=\sin x$, $c=\frac{\pi}{4}$, evaluate

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} = \lim_{h\to 0}\frac{\sin(\frac{\pi}{4}+h)-\sin(\frac{\pi}{4})}{h}$$

$$\sin(\tfrac{\pi}{4}+h) = \sin\tfrac{\pi}{4}\cos(h)+\cos\tfrac{\pi}{4}\sin(h) = \tfrac{\sqrt{2}}{2}\cos(h)+\tfrac{\sqrt{2}}{2}\sin(h)$$

$$= \lim_{h\to 0}\frac{\frac{\sqrt{2}}{2}\cos(h)+\frac{\sqrt{2}}{2}\sin(h)-\frac{\sqrt{2}}{2}}{h}$$

$$= \lim_{h\to 0}\left[\frac{\frac{\sqrt{2}}{2}(\cos(h)-1)}{h}+\frac{\frac{\sqrt{2}}{2}\sin(h)}{h}\right]$$

$$= 0\cdot\frac{\sqrt{2}}{2}+1\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

For 6, 7, 8, find $f'(x)$ by $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Given.

6. $f(x) = 3x+2$, then $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

6. (Section 2.1, Problem 4)

$$= \lim_{h\to 0}\frac{3(x+h)+2-(3x+2)}{h} = \lim_{h\to 0}\frac{3x+3h+2-3x-2}{h}$$

$$= \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = 3$$

7. Given $f(x) = 1-x^2$, then

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1-(x+h)^2-(1-x^2)}{h}$$

7. (Section 2.1, Problem 8)

$$= \lim_{h\to 0}\frac{1-(x^2+2xh+h^2)-1+x^2}{h} = \lim_{h\to 0}\frac{-2xh-h^2}{h}$$

$$= \lim_{h\to 0} -2x-h = -2x$$

8. Given $f(x) = \sqrt{x+1}$, then

8. (Section 2.1, Problem 15)

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$= \lim_{h\to 0}\frac{(x+h+1)-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \lim_{h\to 0}\frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

9. Compare $\lim\limits_{h\to 0}\frac{(-3+h)^4-81}{h}$ with $\lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}$

We have $f(c+h) = (-3+h)^4$ and $f(c) = 81$

9. (Section 2.1, Problem 18)

which means $f(x) = x^4$ and $c = -3$.

10. Given $f(x) = \begin{cases} 8+x^3, & x \le 1 \\ Bx+C & x > 1 \end{cases}$

Check f is differentiable everywhere

10. (Section 2.1 Problem 28)

We only need to check f as $x = 1$.

Since "f is differentiable implies f is continuous".

So first we check continuity (①=②=③)

then check the existence of derivative of f

by the definition of derivative $\left(\lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}\right)$

I. continuity.

① $\lim\limits_{x\to 1^+} f(x) = B+C$. ② $\lim\limits_{x\to 1^-} f(x) = 8+1 = 9$, ③ $f(1) = 9$

①=②=③ $\Rightarrow$ $\boxed{B+C=9}$ —(*)

II. differentiability

$\triangle$ $\lim\limits_{h\to 0^+}\frac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0^+}\frac{B(1+h)+C-9}{h} = \lim\limits_{h\to 0^+}\frac{Bh+\overbrace{B+C-9}^{\text{by (*)}\ 0}}{h}$

$= \lim\limits_{h\to 0^+}\frac{Bh}{h} = B$

$\triangle$ $\lim\limits_{h\to 0^-}\frac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0^-}\frac{8+(1+h)^3-9}{h} = 3$

$\Rightarrow B = 3$ and $\overset{\text{by (*)}}{C = 6}$