Continuity of $f(x)$ @ $x=c$.

check ① $\lim_{x\to c^+} f(x)$, ② $\lim_{x\to c^-} f(x)$ and ③ $f(c)$.

Case 1

① = ② = ③, ①②③ exist

f is conti @ $x=c$

Case 2

① = ② ≠ ③, ①② exist

f is disconti. and Removable.

Case 3

① ≠ ②, ①② exist

f has a jump discontinuity @ $x=c$

Case 4

①, ② DNE.

f has an infinite disconti. @ $x=c$

Math 1431, Section 17699

Homework 2 (10 points)

Due 2/5 in Recitation

Name: Sol PSID:

Instructions:

- print your name clearly.
- always show your work to get full credit;
- staple all the pages together in the right order.
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date.

1. (Section 1.4, Problem 6) $f(x) = |25-x^2|$ @ $x=-5$

check ① $\lim_{x\to -5^+} f(x)$, ② $\lim_{x\to -5^-} f(x)$ and ③ $f(-5)$

① $= |25-(-5)^2| = 0$, ② $= |25-(-5)^2| = 0$, ③ $= 0$

Since ① = ② = ③, then f is continuous at $x=-5$

2. (Section 1.4, Problem 8)

Given $f(x) = \begin{cases} -2x^2+1 & x<-1 \\ 2 & x=-1 \\ x^3 & x>-1 \end{cases}$, @ $x=-1$.

check ① $\lim_{x\to -1^+} f(x)$, ② $\lim_{x\to -1^-} f(x)$ and ③ $f(-1)$

① $= (-1)^3 = -1$, ② $= -2(-1)^2+1 = -1$, ③ $= 2$.

① = ② ≠ ③, then f is discontinuous at $x=-1$ and it's a Removable one.

3. (Section 1.4, Problem 10)

Given $f(x) = \begin{cases} \frac{|x-1|}{x-1} & x\neq 1 \\ 0 & x=1 \end{cases}$ @ $x=1$

check ① $\lim_{x\to 1^+} f(x)$, ② $\lim_{x\to 1^-} f(x)$ and ③ $f(1)$

① $= 1$, ② $= -1$, ③ $= 0$

① ≠ ② ≠ ③ and ①, ② exist

$f(x)$ is discontinuous at $x=1$ and it's a jump one.

4. (Section 1.4, Problem 20)

Given $f(x) = \frac{x+2}{x^2-3x-10}$, check discontinuity.

it may happen when denominator equals zero.

$\Rightarrow x^2-3x-10=0 \Rightarrow (x-5)(x+2)=0 \Rightarrow x=5$ or $x=-2$

Case 1 @ $x=-2$

$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{(x+2)}{(x+2)(x-5)} = \frac{1}{-7}$ exist but $f(-2)$ DNE

A Removable discontinuity.

(i.e ① = ② ≠ ③)

Case 2 @ $x=5$

$\lim_{x\to 2} f(x)$ DNE f has an infinite dis.

5. (Section 1.4, Problem 26)

$f(x) = \begin{cases} -4x+5 & x<1 \\ 0 & x=1 \\ \frac{1}{x^2} & x>1 \end{cases}$ → check "$x=1$"

$\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1^-} f(x)$, $f(1)$

$\frac{1}{1^2} = 1$, $-4\cdot 1+5 = 1$, 0

①②③ exist

① ≠ ② ≠ ③

A jump discontinuity

Given $f(x) = \dfrac{x^2+7x+10}{x+5}$, check the continuity at $x=-5$

$$\lim_{x\to -5} f(x) = \lim_{x\to -5} \frac{x^2+7x+10}{x+5} = \lim_{x\to -5} \frac{(x+5)(x+2)}{(x+5)} = \lim_{x\to -5}(x+2) = -3$$

6. (Section 1.4, Problem 34)

that is, ① = ② but ③ DNE.

To Redefine ③ ($f(-5)$), we let $f(-5)=-3$,

then ① = ② = ③ which means this redefined f is continuous everywhere.

7. (Section 1.5, Problem 2)

Given $f(x) = \begin{cases} Ax-B & x\le 1 \\ -24x & 1<x<5 \\ Bx^2-A & x\ge 5 \end{cases}$, Find A, B s.t. f is conti. everywhere.

as $x=1$

① $\lim_{x\to 1^+} f(x) = -24$

② $\lim_{x\to 1^-} f(x) = A-B$

③ $f(1) = A-B$

If ① = ② = ③

$\Rightarrow A-B=-24$.

$25B-A=-120 \Leftarrow$

$\Rightarrow B=-6,\ A=-30$

as $x=5$

① $\lim_{x\to 5^+} f(x) = 25B-A$

② $\lim_{x\to 5^-} f(x) = -24\cdot 5 = -120$

③ $f(5) = 25B-A$

8. (Section 1.5, Problem 10)

8. Given $f(x) = \sqrt{3x^2-6x}-2$ on $[3,5]$

check $f(3)$ and $f(5)$ to see 0 is between them or NOT.

$f(3) = 3-2 = 1,\ f(5) = 5-2 = 3$

Since $f(3)>0,\ f(5)>0$, IVT fails.

9. (Section 1.5, Problem 14)

9. Solving $\dfrac{1}{x-2} + \dfrac{3}{4x-36} \ge 0$

First $x\ne 2$ and $4x-36\ne 0 \Rightarrow \boxed{\begin{matrix} x\ne 2 \\ x\ne 9\end{matrix}}$

Then $\dfrac{1}{x-2} + \dfrac{3}{4(x-9)} = \dfrac{4(x-9)+3(x-2)}{(x-2)(4x-36)}$

$= \dfrac{7x-42}{(x-2)4(x-9)} \ge 0 \Leftrightarrow 7(x-6)(x-2)\cdot 4(x-9) \ge 0$

− + − +

2 6 9

$\Rightarrow 2<x\le 6$ or $x>9$.

10. (Section 1.5, Problem 30)

Given $f(x) = \dfrac{x^2+x}{x-1}$ on $[\frac{5}{2}, 4]$

Find c such that $f(c)=6$.

First, check IVT works.

$f(\frac{5}{2}) = \frac{35}{6},\ f(4) = \frac{20}{3} \Rightarrow f(\frac{5}{2}) < 6 < f(4)$ ✓

Then, find $c\in(\frac{5}{2}, 4)$ s.t. $f(c)=6$

$\Rightarrow \dfrac{x^2+x}{x-1} = \dfrac{6}{1} \Rightarrow x^2+x = 6(x-1)$

$\Rightarrow x^2-5x+6=0$

$\Rightarrow (x-2)(x-3)=0 \Rightarrow x=2$ or $x=3$

$\Rightarrow x=3$