Math 1431 – Section 17699

Homework 1 (10 points)

Due Thurs. 1/29 in Recitation

Name: Sol. PSID:

Instructions:

- Print out this file and complete the problems.
- If the problem is from the text, the section number and problem number are in parentheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.

---

1. Find the following limits:

a. $\lim_{h\to 0}\frac{(3+h)^2-9}{h}$ $= \lim_{h\to 0}\frac{(3+h)^2-3^2}{h} = \lim_{h\to 0}\frac{(3+h+3)(3+h-3)}{h}$

$\left(* \; a^2-b^2 = (a+b)(a-b)\right)$

$= \lim_{h\to 0}\frac{(6+h)\cdot h}{h} = \lim_{h\to 0} 6+h = 6.$

b. $\lim_{x\to 2}\frac{x}{x^2-4}$ $=$ DNE

As $x\to 2^+$, $x^2-4 \to$ A postive number which closed to 0

$x\to 2^-$ x^2-4 tends to a negative number which closed to 0

$\Rightarrow \lim_{x\to 2^+}\frac{x}{x^2-4} = \infty$ (unbounded) and $\lim_{x\to 2^-}\frac{x}{x^2-4} = -\infty$ (unbounded) $\Rightarrow$ DNE

c. $\lim_{t\to -4}\left(\frac{3t}{t+4}+\frac{8}{t+4}\right)$ D.N.E.

$= \lim_{t\to -4}\frac{3t+8}{t+4}$

$\lim_{t\to -4^+}\frac{3t+8}{t+4} = -\infty$ (unbounded).

and $\lim_{t\to -4^-}\frac{3t+8}{t+4} = \infty$ (unbounded) $\Rightarrow$ D.N.E.

2. (Section 1.3, Problem 6) $\lim_{x\to 0}\left(\frac{6x^2-7x}{x}\right)$

$$= \lim_{x\to 0} \frac{x(6x-7)}{x}$$

$$= \lim_{x\to 0} 6x-7 = -7$$

3. (Section 1.3, Problem 12) $\lim_{x\to 1} \frac{6x^4-6}{2x-2} = \lim_{x\to 1} \frac{6(x^4-1)}{2(x-1)}$

$$\boxed{* \quad (x^4-1) = (x-1)(x^3+x^2+x+1)}$$

$$= \lim_{x\to 1} \frac{3(x-1)(x^3+x^2+x+1)}{(x-1)}$$

$$= \lim_{x\to 1} 3(x^3+x^2+x+1) = 3\cdot 4 = 12$$

4. (Section 1.3, Problem 16)

$\lim_{x\to 3} \frac{1}{x-3}$ D.N.E.

Since $\lim_{x\to 3^+} \frac{1}{x-3} = \infty$ (unbounded) and $\lim_{x\to 3^-} \frac{1}{x-3} = -\infty$ (unbounded)

$\Rightarrow \lim_{x\to 3} \frac{1}{x-3}$ DNE.

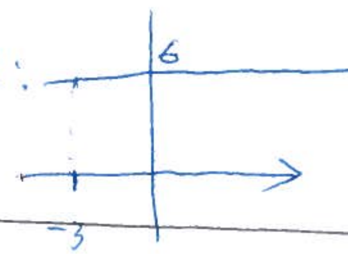
5. (Section 1.3, Problem 18) $\lim\limits_{x\to 8^+} \frac{\sqrt{x-8}}{x} = \frac{0}{8} = 0$

$x \to 8^+ \Rightarrow x > 8$ but pretty close to 8

(It's well-defined to ask the value is nonnegative when we want to find the square root of this value)

6. (Section 1.3, Problem 20) $\lim\limits_{x\to -3} 6 = 6$

(Because 6 is a constant function:



)

$\lim\limits_{x\to\infty} \frac{3x^3+x+6}{2x^2-3} = \infty$ since $\deg(3x^3+x+6) = 3 > 2 = \deg(2x^2-3)$

7. (Section 1.3, Problem 32)

~~Evaluate $\lim\limits_{x\to -3} f(x)$, given that $f(x) = \begin{cases} 6x & x<-3 \\ -18x & x>-3 \end{cases}$~~

~~A: $\lim\limits_{x\to -3^+} f(x) = -18\cdot(-3) = 54$ and $\lim\limits_{x\to -3^-} f(x) = 6\cdot(-3) = -18$~~

~~Using the case "$x>-3$"~~ ~~using the case "$x<-3$"~~

~~Since $\lim\limits_{x\to -3^+} f(x) \neq \lim\limits_{x\to -3^-} f(x) \Rightarrow \lim\limits_{x\to -3} f(x)$ D.N.E.~~

8. (Section 1.3, Problem 34)

Evaluate $\lim_{x\to 0} f(x)$, given that $f(x)=\begin{cases} x^2, & x<0 \\ x-7, & x>0 \end{cases}$

Check $\lim_{x\to 0^+} f(x) = 0-7=-7$ and $\lim_{x\to 0^-} f(x) = 0$

"$x>0$" "$x<0$"

$\Rightarrow \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$, so $\lim_{x\to 0} f(x)$ D.N.E.

9. (Section 1.3, Problem 40)

For $\lim_{x\to 3} \frac{x}{6} = \frac{1}{2}$. Given $\varepsilon = 0.01$. Find max δ.

let $f(x)=\frac{x}{6}$, $L=\frac{1}{2}$, $a=3$, so by def. of limit,

we have $|f(x)-L|<\varepsilon \Rightarrow \left|\frac{x}{6}-\frac{1}{2}\right|<0.01$.

Try to find a δ s.t. $|x-3|<\delta$,

Since $\left|\frac{x}{6}-\frac{1}{2}\right|<0.01 \Rightarrow |x-3|<0.06 \Rightarrow \delta_{max}=0.06$.

times "6" on both sides

10. (Section 1.3, Problem 43)

Given $f(x)=x^2-4x$, find $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^2-4x-(-3)}{x-1}$

$= \lim_{x\to 1} \frac{x^2-4x+3}{x-1} = \lim_{x\to 1} \frac{(x-1)(x-3)}{(x-1)} = \lim_{x\to 1}(x-3) = -2$.