If $F(x)=\int_a^x f(t)dt$, by F.T.C. $F'(x)=f(x)$

If $F(x)=\int_a^{u(x)} f(t)dt$, then $F'(x)=u'(x)\cdot f(u(x))$.

Math 1431, Section 17699

Homework 13 (10 points)

Due 4/30 in Recitation

Name: Sol PSID:

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 6.2, Problem 2) Given $F(x)=\int_x^0 \frac{dt}{t+5}=-\int_0^x \frac{dt}{t+5}$.

then, by F.T.C. $F'(x)=-\frac{1}{x+5}$. so.

(a) $F'(-1)=-\frac{1}{-1+5}=-\frac{1}{4}$, (b) $F'(0)=-\frac{1}{5}$

(c) $F'(2)=-\frac{1}{2+5}=-\frac{1}{7}$ (d) $F''(x)=+\frac{1}{(x+5)^2}$

2. (Section 6.2, Problem 8) Given $F(x)=\int_0^{\cos(x)}(t^2+4)dt$.

then. $F'(x)=-\sin(x)\cdot[\cos^2(x)+4]$.

(a) $F'(-1)=-\sin(-1)[\cos^2(-1)+4]$ (b) $F'(0)=-\sin(0)\cdot[\cos^2(0)+4]$

$=0\cdot[1+4]=0$.

(c) $F'(2)=-\sin(2)[\cos^2(2)+4]$.

(d) $F''(x)=-\cos(x)[\cos^2(x)+4]-\sin(x)[-2\cos(x)\sin(x)]$

product rule

3. (Section 6.2, Problem 16) $\int_1^2 \frac{1}{x}dx=\ln|x|\Big|_1^2$

$=\ln|2|-\ln|1|$

$=\ln 2-0=\ln 2$.

4. (Section 6.2, Problem 19) $\frac{d}{dx}\left[\int_{4x}^{x^2+1} 10t\, dt\right]$

$=\frac{d}{dx}\left[\int_c^{x^2+1} 10t\,dt-\int_c^{4x} 10t\,dt\right]$

$=(x^2+1)'\cdot 10(x^2+1)-(4x)'10\cdot(4x)$

$=2x\cdot 10\cdot(x^2+1)-40\cdot(4x)$

$=40x^3-140x$

5. (Section 6.3. Problem 26)

$$\int\left(\frac{2x+1}{\sqrt{x}}+\frac{1}{1+x^2}\right)dx=\int\left(2\sqrt{x}+\frac{1}{\sqrt{x}}+\frac{1}{1+x^2}\right)dx.$$

$$=2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\arctan(x)+C.$$

$$=\frac{4}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+\arctan(x)+C.$$

6. (Section 6.3. Problem 40)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(4\cos(x)+10\sin(x))dx.$$

$$=\left[4\sin(x)-10\cos(x)\right]\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}.$$

$$=\left[4\sin(\tfrac{\pi}{3})-4\sin(\tfrac{\pi}{6})\right]-10\left[\cos(\tfrac{\pi}{3})-\cos(\tfrac{\pi}{6})\right]$$

$$=\left[4\cdot\frac{\sqrt{3}}{2}-4\cdot\frac{1}{2}\right]-10\left[\frac{1}{2}-\frac{\sqrt{3}}{2}\right].$$

$$=2\sqrt{3}-2-5+5\sqrt{3}$$

$$=7\sqrt{3}-7.$$

7. (Section 6.3. Problem 56)

$$\int_1^e\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$$

$$=\left[\ln|x|+\frac{1}{x}\right]\Big|_1^e$$

$$=\left[\ln|e|-\ln|1|\right]+\left[\frac{1}{e}-\frac{1}{1}\right]$$

$$=1-0+\frac{1}{e}-1=\frac{1}{e}$$

8. (Section 6.3. Problem 82)

Given $f''(x)=2\sin(x)$, $f'(\frac{\pi}{2})=4$, $f(\pi)=5$.

① $f'(x)=\int 2\sin(x)\,dx=-2\cos(x)+C_1$.

$f'(\frac{\pi}{2})=4 \Rightarrow 4=f'(\frac{\pi}{2})=-2\cos(\frac{\pi}{2})+C_1=C_1$.

$\Rightarrow C_1=4$.

$f'(x)=-2\cos(x)+4$.

② $f(x)=\int f'(x)dx=\int[-2\cos(x)+4]dx$

$=-2\sin(x)+4x+C_2$.

$f(\pi)=5 \Rightarrow 5=f(\pi)=-2\sin(\pi)+4\pi+C_2$

$=4\pi+C_2$.

$\Rightarrow C_2=5-4\pi$.

$\Rightarrow f(x)=-2\sin(x)+4x+5-4\pi$.

9. (Section 6.4, Problem 34)

$$\int \frac{\sin(x)}{\sqrt{2+\cos(x)}}\,dx. = \int \frac{-du}{\sqrt{u}} = -2u^{\frac{1}{2}} + C.$$

$$= -2(2+\cos(x))^{\frac{1}{2}} + C.$$

Let $u = 2+\cos(x)$

$\Rightarrow du = -\sin(x)dx$

$\Rightarrow -du = \sin(x)dx$

10. (Section 6.4, Problem 48)

$$\int \frac{e^{2x}}{(e^{2x}+1)^2}\,dx. = \int \frac{1}{u^2}\cdot\frac{du}{2} = \frac{1}{2}\int\frac{du}{u^2}.$$

$$= \frac{1}{2}\cdot\left(-\frac{1}{u}\right) + C.$$

$$= -\frac{1}{2}\cdot\frac{1}{e^{2x}+1} + C.$$

Let $u = e^{2x}+1.$

$du = 2e^{2x}dx$

$\Rightarrow \frac{du}{2} = e^{2x}dx$