Indeterminate Form: $\frac{0}{0}, \frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, $0^0, 1^\infty, \infty^0$

Math 1431, Section 17699

Homework 12 (10 points)

Due 4/23 in Recitation

Name: Sol PSID:

Instructions:
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

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1. (Section 5.3, Problem 1) Given $\frac{\ln(x)-1}{x-1}$ and $c=1$.

We have $\frac{\ln(1)-1}{1-1} = \frac{-1}{0}$

which is NOT an indeterminate form.

2. (Section 5.3, Problem 9) $\lim\limits_{x\to 0} \frac{\sin^2 x}{x^2} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim\limits_{x\to 0} \frac{\cancel{2}\sin(x)\cos(x)}{\cancel{2}x}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim\limits_{x\to 0} \frac{\cos^2(x)-\sin^2(x)}{1} = 1.$

$$\boxed{\begin{array}{l}\text{Recall. } \lim\limits_{x\to 0} \frac{\sin^2 x}{x^2} = \lim\limits_{x\to 0} \frac{\sin(x)}{x}\cdot\frac{\sin(x)}{x} = 1\cdot 1 = 1 \\ \left(\lim\limits_{x\to 0} \frac{\sin(ax)}{ax} = 1\right)\end{array}}$$

3. (Section 5.3, Problem 10) $\lim\limits_{x\to 0} \frac{\sin(5x)}{x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim\limits_{x\to 0} \frac{5\cos(5x)}{1} = 5\cdot 1 = 5$

$$\boxed{\text{Recall } \lim\limits_{x\to 0} \frac{\sin(5x)}{x} = \lim\limits_{x\to 0} \frac{\sin(5x)}{5x}\cdot 5 = 5}$$

4. (Section 5.3, Problem 18) $\lim\limits_{x\to 0} \frac{e^x+e^{-x}-2}{1-\cos(2x)} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim\limits_{x\to 0} \frac{e^x-e^{-x}}{2\sin(2x)}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim\limits_{x\to 0} \frac{e^x+e^{-x}}{4\cos(2x)} = \frac{e^0+e^0}{4} = \frac{2}{4} = \frac{1}{2}$

5. (Section 5.3, Problem 24)

$$\lim_{x\to 0}\frac{\arcsin(x)}{2x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}}{2} = \frac{1}{2}.$$

6. (Section 5.3, Problem 28)

$$\lim_{x\to 0}\frac{\arctan(x)}{\arctan(2x)} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{\frac{1}{1+(2x)^2}} = 1$$

7. (Section 5.3, Problem 47)

$$\lim_{x\to\infty} x^2\cdot\sin\left(\tfrac{1}{x}\right) \overset{(\infty\cdot 0)}{=} \lim_{x\to\infty}\frac{\sin(\frac{1}{x})}{(\frac{1}{x^2})}$$

[use "$\frac{1}{\infty}=0$"]

$$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{\frac{-1}{x^2}\cos(\frac{1}{x})}{\frac{-2}{x^3}} = \lim_{x\to\infty}\frac{\cos(\frac{1}{x})}{\frac{2}{x}}$$

$$= \lim_{x\to\infty}\frac{x}{2}\cos\left(\tfrac{1}{x}\right) \text{ DNE.}$$

(since $\cos(\frac{1}{x})\to 1$ but $\frac{x}{2}\to\infty$ as $x\to\infty$)

8. (Section 5.3, Problem 76)

$$\lim_{x\to\infty}(\sqrt{x}+1)^{\frac{1}{\sqrt{x}}} \overset{(\infty^0)}{=} \lim_{x\to\infty} e^{\ln(\sqrt{x}+1)^{\frac{1}{\sqrt{x}}}}$$

$f(x) = e^{\ln(f(x))}$

$$= \lim_{x\to\infty} e^{\frac{1}{\sqrt{x}}\ln(\sqrt{x}+1)}$$

$$= e^{\lim_{x\to\infty}\frac{\ln(\sqrt{x}+1)}{\sqrt{x}}} = e^0 = 1$$

Since exp. function is continuous

and $\lim_{x\to\infty}\frac{\ln(\sqrt{x}+1)}{\sqrt{x}} \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{\frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}+1}}{\frac{1}{2\sqrt{x}}}$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{x}+1} = 0$$

9. Given $f(x)=3x-x^2$
$x\in[0,3]$, $n=3$.

Partition	length of partition	Value of left endpoint	Value of right endpoint	value of midpoints
$[0,1]$	1	$f(0)=0$	$f(1)=2$	$f(\frac{1}{2})=\frac{5}{4}$
$[1,2]$	1	$f(1)=2$	$f(2)=2$	$f(\frac{3}{2})=\frac{9}{4}$
$[2,3]$	1	$f(2)=2$	$f(3)=0$	$f(\frac{5}{2})=\frac{5}{4}$

9. (Section 6.1, Problem 18)

Riemann Sum

$= \sum\left[(\text{length of partition}) \times (\text{Value of \_\_ point})\right] \Rightarrow$ (a) $1\cdot 0+1\cdot 2+1\cdot 2=4.$

(b) $1\cdot 2+1\cdot 2+1\cdot 0=4$

(c) $1\cdot\frac{5}{4}+1\cdot\frac{9}{4}+1\cdot\frac{5}{4}=\frac{19}{4}$

10. (Section 6.1, Problem 22)

Given $\int_0^5 f(x)dx=8$. $\int_2^5 f(x)dx=7$, $\int_2^9 f(x)dx=-4$, $\int_0^5 g(x)\,dx=10$.

(a) $\int_0^5[2f(x)-4g(x)]dx=2\int_0^5 f(x)dx-4\int_0^5 g(x)dx=2\cdot 8-4\cdot 10=-24.$

(b) $\int_0^2 f(x)dx=\int_0^5 f(x)dx-\int_2^5 f(x)dx=8-7=1.$

(c) $\int_9^5 f(x)dx=-\int_5^9 f(x)\,dx=-\left[\int_2^9 f(x)dx-\int_2^5 f(x)dx\right]=-[-4-7]=11.$

(d) $\int_5^0 6g(x)dx=-6\int_0^5 g(x)dx=-6\cdot(10)=-60$