Math 1431, Section 17699

Homework 11 (10 points)

Due 4/16 in Recitation

Name: Sol PSID:

Instructions:

- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it.
- submit the completed assignment to your Teaching Assistant in lab on the due date

1. (Section 5.1, Problem 1) Given $x+y=100$, Find the greatest value of xy.

Since $x+y=100 \Rightarrow y=100-x$, then $xy=x(100-x)$

Let $f(x)=x(100-x)$, then $f'(x)=-2x+100$

$= 100x-x^2$ check number line of f'

$\Rightarrow f'(x)=0$ implies $x=50$.

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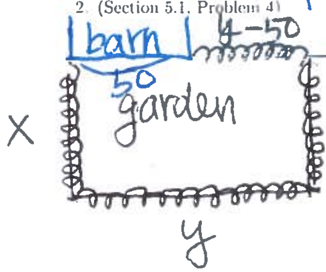
which has max of f $\Rightarrow x=50, y=50$

2. (Section 5.1, Problem 4)



"~~~": fencing material.

Let the width be x feet, length be y feet

"Barn is 50 feet long" implies $y-50\geq 0 \Rightarrow y\geq 50$

"400 feet fencing material" implies $2x+y+y-50=400$

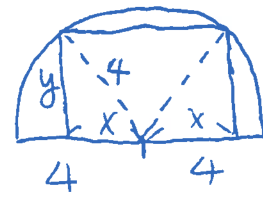
"Find max. area" implies "Find max. value of xy".

$\Rightarrow x=\frac{450-2y}{2}$ implies $xy=y\cdot\frac{450-2y}{2}$ $\Rightarrow$ Let $f(x)=y(225-y)$

$(\because y\geq 50)$ $f'(y)=-2y+225$ $\Rightarrow$ Critical pt: $y=\frac{225}{2}$

50 + + + + $\frac{225}{2}$ − − −

Max. $\Rightarrow y=\frac{225}{2}$, $x=\frac{225}{2}$ has max $\left(\frac{225}{2}\right)^2$



Let the length of one side of rectangle be "2x", and the other one be "y"

- Find largest area $\Rightarrow$ Find max. of $2xy$
- The relation between x and y

$\Rightarrow x^2+y^2=4^2$ (Pythagorean's)

3. (Section 5.1, Problem 8)

$\Rightarrow y=\sqrt{16-x^2}$. $2xy=2x\sqrt{16-x^2}$

Let $f(x)=2x\sqrt{16-x^2}$, $f'(x)=2\sqrt{16-x^2}+\frac{2x}{2}\cdot\frac{1}{\sqrt{16-x^2}}\cdot -2x$

$=\frac{2(16-x^2)-2x^2}{\sqrt{16-x^2}}=\frac{32-4x^2}{\sqrt{16-x^2}}$

$f'(x)=0 \Rightarrow 32-4x^2=0$

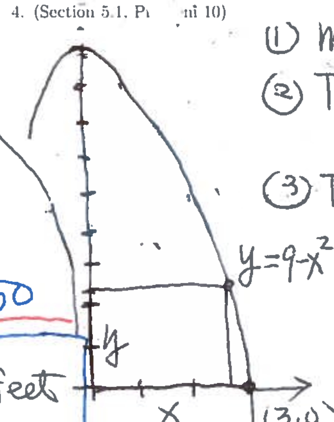
Critical number, $x=\pm 2\sqrt{2}$ $(0<x<4)$

f': 0 + + + + + $2\sqrt{2}$ − 4

local max at $x=2\sqrt{2}$. $\Rightarrow y=2\sqrt{2}$.

max of $2xy=2\cdot 2\sqrt{2}\cdot 2\sqrt{2}=16$

4. (Section 5.1, Problem 10)



① max. function: xy

② The relation between x and y: $y=9-x^2$

③ The restriction of x: $0\leq x\leq 3$

④ $f(x)=xy=x(9-x^2)$

$=-x^3+9x$

$f'(x)=-3x^2+9 \Rightarrow x=\sqrt{3}, \cancel{-\sqrt{3}}$

0 + + + + $\sqrt{3}$ − − 3

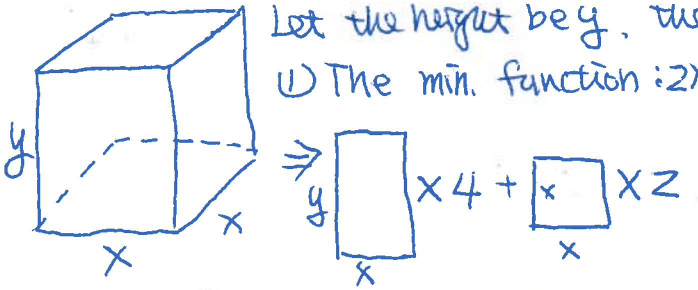
as $x=\sqrt{3}$ $f(x)$ has max.

$\Rightarrow x=\sqrt{3}$, $y=9-(\sqrt{3})^2=6$

$\Rightarrow$ max of $xy=6\sqrt{3}$.

Let the height be y, the side of base be x

① The min. function: $2x^2+4xy$



$\Rightarrow$ $\times 4 + \times 2$

② The relation: $x^2y=4000$

$\Rightarrow y=\frac{4000}{x^2}$

5. (Section 5.1, Problem 16)

③ The restriction: $x>0,\ y>0$.

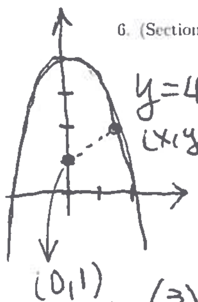
④ $f(x)=2x^2+4xy=2x^2+4x\cdot\frac{4000}{x^2}=2x^2+\frac{16000}{x}$

$f'(x)=4x-\frac{16000}{x^2}=\frac{4x^3-16000}{x^2}$ $\Rightarrow$ critical, $x=\sqrt[3]{4000}$

$\Rightarrow$ As $x=\sqrt[3]{4000}$, $y=\frac{4000}{(\sqrt[3]{4000})^2}$

$2x^2+4xy$ has min.

6. (Section 5.1, Problem 18)



① The min. function means the min. distance from $(0,1)\to(x,y)$

$\Rightarrow d^2=(y-1)^2+x^2$.

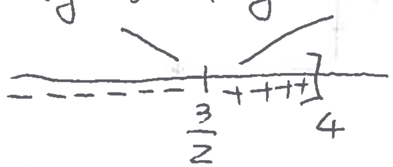
② The relation: $y=4-x^2 \Rightarrow x^2=4-y$

③ The restriction: $x\in\mathbb{R},\ y\le 4$.

④ $f(y)=(y-1)^2+x^2=(y-1)^2+4-y$

$=y^2-3y+5$.

$f'(y)=2y-3$



$\Rightarrow$ critical: $y=\frac{3}{2}$ $\Rightarrow$ As $y=\frac{3}{2}$, $f(y)$ has min. $\Rightarrow \sqrt{f(y)}$ has min.

$\Rightarrow$ As $y=\frac{3}{2}$, $x=\pm\sqrt{\frac{5}{2}}$, $\sqrt{(y-1)^2+x^2}$ has min.

7. ① The max function: $C(t)=\frac{2t}{4+t^2}$

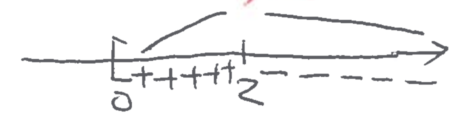
② The restriction: $t\ge 0$,

③ $C'(t)=\frac{2(4+t^2)-2t(2t)}{(4+t^2)^2}=\frac{8-2t^2}{(4+t^2)^2}$

7. (Section 5.1, Problem 27)

$\Rightarrow$ critical number: $t=2$ or $\cancel{-2}$

Number line of C'



$C(t)$ has max at $t=2$.

Formula of differential:

$f(a+h)-f(a)\approx f'(a)\cdot h$

Approximation: $f(a+h)\approx f(a)+f'(a)\cdot h$

8. (Section 5.2, Problem 2)

Given $\sqrt{63}$.

① Find $f(x)$: Let $f(x)=\sqrt{x}$, $f'(x)=\frac{1}{2\sqrt{x}}$

② Pick up a: find 'a' such that a is closed to 63 and $\sqrt{a}$ is integer. $\Rightarrow a=64$.

③ find h: $64+h=63 \Rightarrow h=-1$.

④ Approximation: $\sqrt{63}=f(a+h)\approx f(a)+f'(a)h$

$=\sqrt{64}+\frac{1}{2\sqrt{64}}\cdot(-1)$

$=8+\frac{-1}{2\cdot 8}=\frac{127}{16}$

* Always use radians when computing differentials for trig. function.

9. Given $\sin(58^\circ)$. $58 \cdot \frac{\pi}{180} = \frac{58\pi}{180}$

(1) Find $f(x)$: Let $f(x)=\sin(x)$, $f'(x)=\cos(x)$.

9. (Section 5.2, Problem 14)

(2) Pick up a: a have to be closed to $\frac{58\pi}{180}$ and $\sin(a)$ is easier to find $\Rightarrow a=\frac{\pi}{3}$.

(3) Get h: $\frac{\pi}{3}+h=\frac{58\pi}{180} \Rightarrow h=-\frac{2\pi}{180}=-\frac{\pi}{90}$.

(4) Approximation: $\sin(58^\circ)=f(a+h) \approx f(a)+f'(a)\cdot h$

$$= \sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)\cdot\left(-\frac{\pi}{90}\right)$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\pi}{90} = \frac{\sqrt{3}}{2} - \frac{\pi}{180}.$$

10. (Section 5.2, Problem 26)

Given $f'(x)=(x^2+7)^{\frac{1}{3}}$, $f(\underset{a}{1})=2$. To estimate $f(0.8)$

Now $a=1$, then $1+h=0.8 \Rightarrow h=-0.2$.

Then $f(0.8)=f(a+h) \approx f(a)+f'(a)h$.

$$= f(1)+f'(1)\cdot(-0.2)$$

$$= 2+\underbrace{(1+7)^{\frac{1}{3}}}_{= 2}\cdot(-0.2) = 2+2(-0.2)$$

$$= 1.6.$$