If u is a function of x, i.e. $u(x)$, then

$$\frac{d}{dx}\left(\sin^{-1}(u(x))\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \qquad \frac{d}{dx}\left[\cos^{-1}(u(x))\right] = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1}(u(x))\right) = \frac{1}{1+u^2} \cdot \frac{du}{dx} \qquad \frac{d}{dx}\left[\sec^{-1}(u(x))\right] = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Math 1431, Section 17699

Homework 10 (10 points)

Due 4/9 in Recitation

Name: Sel PSID: ____________

Instructions
- print your name clearly;
- always show your work to get full credit;
- staple all the pages together in the right order;
- before submission check again that the assignment has your name on it;
- submit the completed assignment to your Teaching Assistant in lab on the due date

---

1. (Section 4.4, Problem 14) Given $f(x) = \sin(2\arccos x)$

Find $f(\tfrac{1}{4}) = \sin(2\arccos(\tfrac{1}{4}))$

Let $\theta = \arccos(\tfrac{1}{4})$ $\Rightarrow \frac{1}{4} = \cos(\theta)$

4, θ, 1, $\sqrt{16-1} = \sqrt{15}$

$$= \sin(2\cdot\theta)$$
$$= 2\sin\theta\cos\theta$$
$$= 2\cdot\frac{\sqrt{15}}{4}\cdot\frac{1}{4} = \frac{\sqrt{15}}{8}$$

2. (Section 4.4, Problem 18) Given $f(x) = \sin^{-1}(x^2+1)$

By the formula above, $(u(x) = x^2+1)$

We have

$$f'(x) = \frac{1\cdot(x^2+1)'}{\sqrt{1-(x^2+1)^2}} = \frac{2x}{\sqrt{1-(x^2+1)^2}}$$

3. (Section 4.4, Problem 20) Given $f(x) = \tan^{-1}(xe^x)$

Then we have $(u(x) = xe^x$

$$f'(x) = \frac{1}{1+(xe^x)^2}\cdot(xe^x)'$$
$$= \frac{e^x + xe^x}{1+(xe^x)^2}$$

4. (Section 4.4, Problem 24) Given $f(x) = \sin^{-1}(\ln(x^4))$
$$= \sin^{-1}(4\ln(x))$$

Then we have ($u(x) = 4\ln(x)$)

$$f'(x) = \frac{1}{\sqrt{1-(\ln(x^4))^2}}\cdot(4\ln(x))' = \frac{4}{x\sqrt{1-(\ln(x^4))^2}}$$

5. (Section 4.4, Problem 28) Given $f(x) = \cos(\arctan(\ln x))$

Then, by chain rule, we have

$$f'(x) = -\sin(\arctan(\ln x))\cdot(\arctan(\ln(x)))'$$
$$= -\frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}\cdot\sin(\arctan(\ln x))$$

6. Given $f(x) = 2+\ln(1+\arctan(4x))$
Find the tangent line of f at $x=0$.

6. (Section 4.4, Problem 38)

Slope: $f'(x) = \frac{1}{1+\arctan(4x)} \cdot (1+\arctan(4x))' = \frac{4}{1+(4x)^2} \cdot \frac{1}{1+\arctan(4x)}$

slope at $x=0$, $f'(0) = \frac{4}{1+0} \cdot \frac{1}{1+0} = 4$ ($\because \arctan(0)=0$)

point $(0, f(0)) = (0, 2+\ln(1+0)) = (0,2)$

line equation: $y-2=4x$.

7. (Section 4.5, Problem 14)

$\sinh(x) = \frac{e^x-e^{-x}}{2}$, $\cosh(x) = \frac{e^x+e^{-x}}{2}$

$(\sinh(x))' = \cosh(x)$, $(\cosh(x))' = \sinh(x)$

7. Given $f(x) = \cosh(x^2)$.

$f'(x) = 2x \cdot \sinh(x^2)$.

8. Given $f(x) = e^x \sinh(x)$ (product)

$f'(x) = e^x \cosh(x) + e^x \sinh(x)$

8. (Section 4.5, Problem 20)

9. Given $f(x) = \frac{\cosh(x)}{1-\sinh(x)}$ (quotient rule)

$f'(x) = \frac{\sinh(x)(1-\sinh(x)) - \cosh(x)(-\cosh(x))}{(1-\sinh(x))^2}$

9. (Section 4.5, Problem 28)

$= \frac{\sinh(x) - (\sinh(x))^2 + (\cosh(x))^2}{(1-\sinh(x))^2}$

$(\cosh(x))^2 - (\sinh(x))^2 = 1$, $= \frac{\sinh(x)+1}{(1-\sinh(x))^2}$

10. (Section 4.5, Problem 40)

Given $y = x^{\cosh(x)}$

Find y' (use log. diff.)

① Take "ln" $\Rightarrow \ln y = \cosh(x) \ln x$.

② do derivative $\Rightarrow \frac{y'}{y} = \sinh(x) \cdot \ln x + \frac{\cosh(x)}{x}$

③ $y' = \left[\sinh(x) \cdot \ln x + \frac{\cosh(x)}{x}\right] x^{\cosh(x)}$.