Math 1431, Section 17699

EMCF 10 (10 points)

Due 4/7 at 11:59pm Sol

Instructions:

- Submit this assignment at http://www.casa.uh.edu under "EMCF" and choose EMCF 10.

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1. Find $f'(0)$ if $f(x) = 3xe^x$.
   a. 0
   b. 1
   c. 2
   d. 3
   e. 4
   f. None of the above.

product

$f'(x) = 3e^x + 3xe^x$

$f'(0) = 3e^0 + 3\cdot 0\cdot e^0 = 3\cdot 1 + 3\cdot 0\cdot 1$
$= 3$

2. Find $f'(0)$ if $f(x) = \frac{6x}{e^x+1}$.
   a. 0
   b. 1
   c. 2
   d. 3
   e. 4
   f. None of the above.

quotient rule

$$f'(x) = \frac{6(e^x+1) - 6x\cdot e^x}{(e^x+1)^2}$$

$$f'(0) = \frac{6\cdot(e^0+1) - 6\cdot 0\cdot e^0}{(e^0+1)^2} = \frac{6\cdot(1+1) - 0}{(1+1)^2} = 3$$

3. Evaluate $f'(1)$ if $f(x) = e^{-\frac{1}{x}}$.
   a. $-2e$
   b. $1/e$
   c. $2/e$
   d. e
   e. $2e$
   f. None of the above.

$$f'(x) = \left(-\frac{1}{x}\right)' e^{-\frac{1}{x}} = \frac{1}{x^2} e^{-\frac{1}{x}}$$

$$f'(1) = 1\cdot e^{-1} = \frac{1}{e}$$

4. Evaluate $f'(x)$ if $f(x) = \sqrt{4+e^{2x}}$.
   a. $\frac{e^{2x}}{\sqrt{4+e^{2x}}}$
   b. $\frac{1}{2\sqrt{2e^{2x}}}$
   c. $\frac{xe^{2x-1}}{\sqrt{4+e^{2x}}}$
   d. e^x
   e. None of the above.

$\sqrt{4+e^{2x}} = (4+e^{2x})^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\cdot(4+e^{2x})^{-\frac{1}{2}}\cdot(4+e^{2x})'$$

$$= \frac{2e^{2x}}{2(4+e^{2x})^{\frac{1}{2}}}$$

$$= \frac{e^{2x}}{(4+e^{2x})^{\frac{1}{2}}}$$

5. Choose the expression equivalent to $\ln\frac{3x^2}{7y}$.
   a. $\frac{\ln 3 + \ln(x^2)}{\ln 7 + \ln y}$
   b. $\ln(3x^2) + \ln(7y)$
   c. $\ln 3 - \ln 7 + 2\ln x - \ln y$
   d. $2\ln(3x) - \ln(7y)$
   e. None of the above.

$= \ln 3 + \ln x^2 - \ln 7 - \ln y$
$= \ln 3 + 2\ln x - \ln 7 - \ln y$

6. $\frac{d}{dx}\left(\ln\sqrt{x^2+4}\right) = \frac{d}{dx}\left(\frac{1}{2}\ln(x^2+4)\right)$
   a. $\frac{x}{\sqrt{x^2+4}}$
   b. $\frac{2x}{\sqrt{x^2+4}}$
   c. $\frac{x}{x^2+4}$
   d. $\frac{1}{x}$
   e. None of the above.

$\ln\sqrt{x^2+4} = \ln(x^2+4)^{\frac{1}{2}}$

$$= \frac{1}{2}\,\frac{2x}{x^2+4} = \frac{x}{x^2+4}$$

7. $\frac{d}{dx}\left[\ln\left((5-x)^6\right)\right] = \frac{d}{dx}\left[6\ln(5-x)\right]$
   a. $\frac{1}{(5-x)^6}$
   b. $\frac{6}{x-5}$
   c. $-6(5-x)^5$
   d. $6(5-x)^5$
   e. None of the above.

$$= 6\cdot\frac{-1}{5-x} = \frac{6}{x-5}.$$

8. $\frac{d}{dx}\left(\ln \frac{x(x^2+2)}{\sqrt{x^3-7}}\right) =$ $\frac{d}{dx}\left[\ln x + \ln(x^2+2) - \frac{1}{2}\ln(x^3-7)\right]$

$$= \frac{1}{x} + \frac{2x}{x^2+2} - \frac{1}{2}\cdot\frac{3x^2}{x^3-7}$$

a. $\frac{x^2+2}{x} + \frac{2x^2}{x^2+2} + \frac{3x^2}{2(x^3-7)}$

b. $\frac{1}{x} + \frac{2x}{x^2+2} - \frac{3x^2}{2(x^3-7)}$

c. $\frac{x^2+2}{x} + \frac{2x^2}{x^2+2} - \frac{3x^2}{2(x^3-7)}$

d. $\frac{1}{x} + \frac{2x}{x^2+2} + \frac{3x^2}{2(x^3-7)}$

e. None of the above.

9. Find y' if $\ln y = xy + y^2$.

Take derivative on both sides, we have

$$\frac{y'}{y} = y + xy' + 2yy' \Rightarrow y' = y^2 + xyy' + 2y^2y'$$

shift y' terms to left hand side $\Rightarrow$ $y' - xyy' - 2y^2y' = y^2 \Rightarrow (1-xy-2y^2)y' = y^2$

$$\Rightarrow y' = \frac{y^2}{1-xy-2y^2}.$$

a. $y' = \frac{y}{1-x-2y}$

b. $y' = \frac{y^2}{1-xy-2y^2}$

c. $y' = \frac{y}{y-x-2y^2}$

d. $y' = \frac{y}{\ln y - x - 2y}$

e. None of the above.

10. If $f(x) = x^{\sin x}$, then $f'(x) = x^{\sin x}\left(\frac{\sin x}{x} - (\cos x)(\ln x)\right)$.

a. True

b. False

$$\ln f(x) = \sin(x)\ln x$$

product rule $\Rightarrow$ $\frac{f'(x)}{f(x)} = \frac{\sin(x)}{x} + \cos(x)\cdot\ln(x)$

$$\Rightarrow f'(x) = x^{\sin(x)}\left[\frac{\sin(x)}{x} + \cos(x)\cdot\ln(x)\right]$$