

Sol

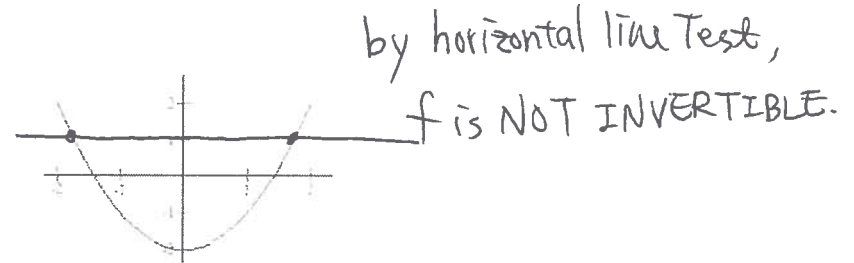
Instructions:

- Submit this assignment at <http://www.casa.uh.edu> under "EMCF" and choose EMCF 9.

1. The function  $f(x) = -2x + 3$  is invertible. Give  $f^{-1}(3)$ . **Let  $x = f^{-1}(3) \Rightarrow f(x) = 3$ .**  
 That is, find a "x" such that  $f(x) = 3$ .  
 $-2x + 3 = 3 \Rightarrow x = 0$ .
- a. -1
  - b. 0
  - c. 1/2
  - d. 1
  - e. None of the above.

2. The function  $f(x) = x^3 + x$  is invertible. Give the slope of the tangent line to the graph of  $f^{-1}(x)$  at  $x = 2$ . **If  $f$  is invertible,  $(f^{-1})'(b) = \frac{1}{f'(a)}$  for  $f(a) = b$ .**  
 at  $x = 2$ ,  $(f^{-1})'(2) = \frac{1}{f'(x)} \Big|_{x=2} = \frac{1}{3x^2 + 1} \Big|_{x=2} = \frac{1}{13}$
- a. -1/2
  - b. 1/13
  - c. 1/2
  - d. 1/4
  - e. 2
  - f. None of the above.

3. The function  $f(x) = x^3 + x$  is invertible. Give  $f^{-1}(2)$ . **Let  $x = f^{-1}(2) \Rightarrow f(x) = 2$**   
 $f(x) = x^3 + x = 2$   
 $\Rightarrow x^3 + x - 2 = 0$   
 $\Rightarrow (x-1)(x^2 + x + 2) = 0$   
 $\Rightarrow x = 1$
- a. -1
  - b. -1/2
  - c. 1/2
  - d. 1
  - e. 2
  - f. None of the above.

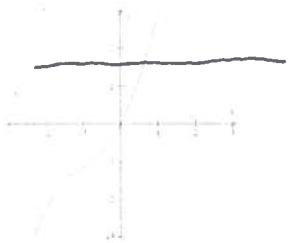


4. The graph of  $f$  is shown above. Determine whether  $f$  is invertible.
- a. The function is invertible
  - b. The function is not invertible

5. The function  $f(x) = -2x + 3$  is invertible. Give the slope of the tangent line to the graph of  $f^{-1}(x)$  at  $x = 2$ .  $(f^{-1})'(b) = \frac{1}{f'(a)} \Big|_{x=2} = \frac{1}{-2} \Big|_{x=2} = -\frac{1}{2}$
- a. -1
  - b. -1/2
  - c. 1/2
  - d. 1
  - e. 2
  - f. None of the above.

6. Determine whether  $f(x) = x^4 + 3x^3 + 1$  is invertible.  $f'(x) = 4x^3 + 9x^2 = x^2(4x+9)$ .  $\Rightarrow$  NOT monotone  $\Rightarrow$  NOT invertible.
- a. The function is invertible
  - b. The function is not invertible

7. The function  $f(x) = x^5 + 3x^3 + x + 1$  is invertible. Give  $(f^{-1})'(1)$ .  $(f^{-1})'(1) = \frac{1}{f'(a)} = \frac{1}{5x^4 + 9x^2 + 1} \Big|_{x=0} = \frac{1}{1} = 1$   
 Find a such that  $f(a) = 1$ .  
 $\Rightarrow a^5 + 3a^3 + a + 1 = 1$   
 $\Rightarrow a = 0$
- a. -2
  - b. -1
  - c. 1/2
  - d. 1
  - e. 2
  - f. None of the above.



By horizontal line test,  
It is invertible.

8. The graph of  $f$  is shown above. Determine whether  $f$  is invertible.

- a. The function is invertible
- b. The function is not invertible

9. Suppose  $(f^{-1})'(1) = 1/3$  and  $f(3) = 1$ . Give the slope of the tangent line to the graph of  $f(x)$  at  $x = 3$ .

- a. -3
- b. -2
- c. 1
- d. 2
- e. 3
- f. None of the above.

$$\frac{1}{3} = (f^{-1})'(1) = \frac{1}{f'(3)} \Rightarrow f'(3) = 3.$$

10. If  $f$  is invertible and  $f(4) = 2$ ,  $f'(4) = 3$ ,  $f'(2) = 9$ , find  $(f^{-1})'(2)$ .

- a.  $1/9$
- b.  $1/4$
- c.  $1/3$
- d.  $1/2$
- e. 1
- f. None of the above.

$$(f^{-1})'(2) = \frac{1}{f'(4)} = \frac{1}{3}.$$