Math 1431, Section 17699

EMCF 7 (10 points)

Due 3/7 at 11:59pm

Sol

Instructions:

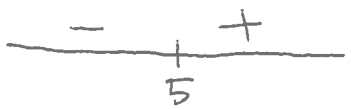
- Submit this assignment at http://www.casa.uh.edu under "EMCF" and choose EMCF 7.

1. The largest interval over which f is increasing for $f(x) = (x-5)^6$ is

a. $[5, \infty)$
b. $[-5, \infty)$
c. $(-\infty, 5]$
d. $(-\infty, -5]$
e. $(-\infty, \infty)$
f. None of the above.

$f'(x) > 0$

$f'(x) = 6(x-5)^5 = 0$



$\Rightarrow x = 5$

$\Rightarrow f'(x) > 0 \Rightarrow x \in [5, \infty)$.

2. $f(x) = \cos(2x)$ has an absolute minimum on $[0, \pi]$ of

a. -1
b. 0
c. 1
d. $\pi/2$
e. 2π
f. None of the above.

it happens as $f'(x) = 0$

$f'(x) = -2\sin(2x) = 0$

$\Rightarrow 2x = 0, \pi, 2\pi$.

$x = 0, \frac{\pi}{2}, \pi$

check $f(0) = 1$

$f(\frac{\pi}{2}) = \cos(\pi) = -1$ ✓

$f(\pi) = \cos(2\pi) = 1$

3. Find the value c that satisfies Rolle's Theorem for $f(x) = \cos(x)$ on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

a. $\pi/4$
b. $\pi/2$
c. $3\pi/4$
d. π
e. 0
f. None of the above.

check $f(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0$, $f(\frac{3\pi}{2}) = \cos(\frac{3\pi}{2}) = 0$

$f(\frac{\pi}{2}) = f(\frac{3\pi}{2})$ ✓

$f'(x) = -\sin(x)$.

$-\sin(c) = f'(c) = 0 \Rightarrow c = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$

(c belongs to $(\frac{\pi}{2}, \frac{3\pi}{2})$)

4. Give the smallest critical number of $f(x) = 5x^4 + 3x^2 + 9$.

a. -1
b. $-\sqrt{\frac{3}{10}}$
c. 0
d. 1
e. 2
f. None of the above.

c such that $f'(c) = 0$ which is in the domain of f.

$f'(x) = 20x^3 + 6x = 2x(10x^2 + 3) = 0$

$x = 0$ ($10x^2 + 3$ is ALWAYS positive)

5. Give the number of values of x where $f(x) = 1 - 8x^2 - x^4$ has a local maximum.

a. 0
b. 1
c. 2
d. 3
e. 4
f. None of the above.

It happens as $f'(x) = 0$ & $f''(x) < 0$

$f'(x) = -16x - 4x^3 = -4x(x^2 + 4) = 0$

$\Rightarrow x = 0$

$f''(x) = -16 - 12x^2 < 0$ as $x = 0$

$f'(x) = -3 + 3x^2 \Rightarrow f'(1) = 0$

6. Classify the critical number $x = 1$ for the function $f(x) = 5 - 3x + x^3$.

a. Local min
b. Local max
c. None of the above.

$f''(x) = 6x \Rightarrow f''(1) > 0$

$\Rightarrow x = 1$ is a local min.

7. Classify the critical number $x = 0$ for the function $f(x) = x^3$.

a. Local min
b. Local max
c. None of the above.

$f'(x) = 3x^2$, $f'(0) = 0$

$f''(x) = 6x$, $f''(0) = 0$

$\Rightarrow$ None of above.

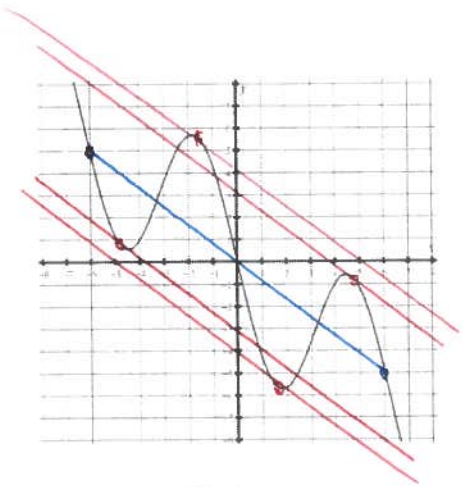
8. Give the number of intervals on which $f(x) = 4x - \frac{4}{3}x^3$ is decreasing.

a. 0
b. 1
c. 2
d. 3
e. 4
f. None of the above.

$f'(x) = 4 - 4x^2 = 4(1+x)(1-x)$

$\Rightarrow 2$

$(-\infty, -1) \cup (1, \infty)$.



4 Points.

9. The graph of f is given above. Give the number of values that satisfy the mean value theorem on the interval $[-6, 6]$.

   a. 0
   b. 1
   c. 2
   d. 3
   e. 4
   f. None of the above.

10. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle is zero?

   a. 1
   b. 2
   c. 3
   d. 4
   e. 5
   f. None of the above.

$\Rightarrow x'(t) = 0 \quad \Rightarrow x'(t) = 2t - 6 = 0$

$\Rightarrow t = 3.$