Math 1431, Section 17699

EMCF 6 (10 points)

Due 2/24 at 11:59pm

Sol

Instructions:

- Submit this assignment at http://www.casa.uh.edu under "EMCF" and choose EMCF 6.

1. If $f(x) = \sqrt{\cos^2(x)+1}$, then $f'(x) = \dfrac{1}{2\sqrt{\cos^2(x)+1}}$.
   a. True
   b. False

   11 $(\cos^2(x)+1)^{\frac{1}{2}}$, $f'(x) = \frac{1}{2}(\cos^2(x)+1)^{-\frac{1}{2}} \cdot 2\cos(x)\cdot[-\sin(x)]$

2. If $z = xy^2$, then $\dfrac{dz}{dt} = 2xy\dfrac{dy}{dt} + y^2\dfrac{dx}{dt}$.
   a. True
   b. False

   $\frac{d}{dt}(xy^2) = \frac{dx}{dt}\cdot y^2 + x\frac{dy^2}{dt} = y^2\frac{dx}{dt} + x\cdot 2y\frac{dy}{dt}$

3. If $z = x^3y$, then $\dfrac{dz}{dt} = 3x^2y\dfrac{dx}{dt}\dfrac{dy}{dt}$.
   a. True
   b. False

   $\frac{d}{dt}(x^3y) = \frac{d}{dt}(x^3)\,y + x^3\frac{dy}{dt} = 3x^2y\frac{dx}{dt} + x^3\frac{dy}{dt}$

4. A spherical balloon is inflating. The rate the volume is changing at $r = 2$ m is given by $\dfrac{dV}{dt} = 16\pi\dfrac{dr}{dt}$.
   a. True
   b. False

   Since $V = \frac{4}{3}\pi r^3$, do "$\frac{d}{dt}$" on both sides, we have

   $\frac{d}{dt}V = \frac{4}{3}\pi\frac{d}{dt}(r^3) = \frac{4\pi}{3}\cdot 3r^2\frac{dr}{dt} \xrightarrow{r=2} \frac{dV}{dt} = \frac{4\pi}{3}\cdot 3\cdot 2^2\frac{dr}{dt} = 16\pi\frac{dr}{dt}$

5. If $f(x) = \sqrt{x}$, then $f'(x) = \dfrac{1}{2\sqrt{x}}$.
   a. True
   b. False

   $x^{\frac{1}{2}}$, $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

6. If $f(x) = \dfrac{1}{x}$, then $f'(x) = \dfrac{-1}{x^2}$.
   a. True
   b. False

   x^{-1}, $f'(x) = -x^{-2} = \frac{-1}{x^2}$

$f(x) = \frac{5}{3}x^4 - 2x^3 + x - \frac{4}{3}x^{-1}$

$f'(x) = \frac{20}{3}x^3 - 6x^2 + 1 + \frac{4}{3}x^{-2}$

7. If $f(x) = \dfrac{5}{3}x^4 - 2x^3 + x - \dfrac{4}{3x}$, then $f'(x) = \dfrac{20}{3}x^3 - 6x^2 + 1 + \dfrac{4}{3x^2}$.
   a. True
   b. False

8. $\lim_{x\to 0}\dfrac{\sin(x)}{x} = 1$
   a. True
   b. False

   $\lim_{x\to 0}\frac{\sin(ax)}{ax} = 1$

9. $\lim_{x\to 0}\dfrac{|x|}{x} = 1$
   a. True
   b. False

   $|x| = \begin{cases} x, & x>0; \\ -x, & x<0, \end{cases}$

   $\Rightarrow \frac{|x|}{x} = \begin{cases} 1, & x>0; \\ -1, & x<0. \end{cases}$

   $\lim_{x\to 0^+}\frac{|x|}{x} = 1$, $\lim_{x\to 0^-}\frac{|x|}{x} = -1$ $\Rightarrow \lim_{x\to 0}\frac{|x|}{x}$ DNE.

10. Find the slope of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point $(3,3)$.
    a. -3
    b. -2
    c. -1
    d. 0
    e. 1
    f. None of these.

    do "$\frac{d}{dx}$":

    $\frac{d}{dx}(x^3+y^3) = \frac{d}{dx}(6xy)$

    $\Rightarrow \frac{d(x^3)}{dx} + \frac{d(y^3)}{dx} = 6y + 6x\frac{dy}{dx}$

    $\Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$

    at $(3,3)$

    $\Rightarrow 3\cdot 3^2 + 3\cdot 3^2\frac{dy}{dx} = 6\cdot 3 + 6\cdot 3\frac{dy}{dx}$

    $\Rightarrow 27 + 27\frac{dy}{dx} = 18 + 18\frac{dy}{dx}$

    $\Rightarrow \frac{dy}{dx} = -1$