Math 1431, Section 17699

EMCF 5 (10 points)

Due 2/21 at 11:59pm

Sol

Instructions:

- Submit this assignment at http://www.casa.uh.edu under "EMCF" and choose EMCF 5.

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1. $f(x) = \frac{1}{x^2-3}$ has a removable discontinuity at $x = \sqrt{3}$.

a. True b. False

$f(x) = \frac{1}{(x-\sqrt{3})(x+\sqrt{3})}$

f at $x=\sqrt{3}$ is a infinite discontinuity

2. $f(x) = \sin(7x)$. $f'(x) =$ $7\cos(7x)$.

chain rule

a. $\cos(7x)$

b. $7\cos(7x)$

c. $-\cos(7x)$

d. $-7\cos(7x)$

e. 0

f. None of these.

3. $y = \sin(\cos(x))$. Find dy/dx. $= \cos(\cos(x)) \cdot [-\sin(x)]$

chain rule

a. $-\sin(\cos(x))\sin(x)$

b. $\cos(\cos(x))$

c. $-\cos(\cos(x))\sin(x)$

d. $-\sin(x)$

e. $\cos(x)$

f. None of these.

4. $y = \tan(\cos(x))$. Find dy/dx. $= \cancel{\sec(\cos(x))\cdot\tan(\cos(x))\cdot[-\sin(x)]}$

chain rule

$= \sec^2(\cos(x)) \cdot [-\sin(x)]$

$= -\sec^2(\cos(x))\,[\sin(x)]$

a. $-\sec^2(\cos(x))\sin(x)$

b. $\sec^2(\cos(x))$

c. $\sin(x)$

d. $-\sin(x)$

e. $\cos(x)$

f. None of these.

5. If $x^2+y^2=25$, find the value of $\frac{dy}{dx}$ at the point $(3,4)$

a. 3/4

b. 0

c. 1

d. 4/5

e. −3/4

f. None of these.

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$.

$\Rightarrow 2x + 2y\cdot\frac{dy}{dx} = 0$.

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

at $(3,4)$ $\left.\frac{dy}{dx}\right|_{(3,4)} = \frac{-3}{4}$

6. If $x^2+xy+y^2=7$, find the value of $\frac{dy}{dx}$ at the point $(1,2)$

a. −3/5

b. −3/4

c. 3/5

d. 4/5

e. −4/5

f. None of these.

$\frac{d}{dx}(x^2+xy+y^2) = \frac{d}{dx}(7)$

$\Rightarrow 2x + y + x\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0$

product rule

$(2x+y)+(x+2y)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$

at $(1,2) \Rightarrow \frac{-(2+2)}{1+4} = -\frac{4}{5}$

7. If $\sqrt{x}+\sqrt{y}=3$, find the value of $\frac{dy}{dx}$ at the point $(1,4)$

a. −1

b. 1

c. 3

d. −2

e. 4

f. None of these.

$\frac{d}{dx}\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right) = \frac{d}{dx}(3)$.

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\cdot\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}} = -\frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{y}}} = -\frac{\sqrt{y}}{\sqrt{x}}$

at $(1,4)$ $\left.\frac{dy}{dx}\right|_{(1,4)} = -\frac{\sqrt{4}}{\sqrt{1}} = -2$

8. Find the slope of the tangent line to the curve $xy^2 + x^2y = 2$ at the point $(1, 1)$.

a. 5
b. −1
c. −3
d. 1
e. −5
f. None of these.

Find $\frac{dy}{dx}$ at $(1,1)$ $\Rightarrow \frac{d}{dx}(xy^2+x^2y) = \frac{d}{dx}(2)$

product rule $\Rightarrow y^2 + x\cdot 2y\cdot\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} = 0$

$(y^2+2xy) + (2xy+x^2)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(y^2+2xy)}{(2xy+x^2)}$

at $(1,1)$. $\left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{1+2}{2+1} = -1.$

9. Let $y = f(x)$. If $xy^2 + xy = 6$ and $f(3) = 1$, find $f'(3)$.

a. −1/3
b. −1
c. 1/5
d. −1/6
e. 8
f. None of these.

Find $f'(3) = \left.\frac{dy}{dx}\right|$ at $x=3$, $y=1$

(I know this from "$f(3)=1$")

$\frac{d}{dx}(xy^2+xy) = \frac{d}{dx}(6)$

product rule $\Rightarrow y^2 + 2xy\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$\Rightarrow (y^2+y) + (2xy+x)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-(y^2+y)}{(2xy+x)}$ $\Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=3\\y=1}} = \frac{-2}{9}$

10. If $x^2 + y^2 = 25$, find an expression for y''.

"$\frac{d^2y}{dx^2}$"

a. $\frac{x^3 - y^3}{y^6}$
b. $\frac{x^3 + y^3}{y^6}$
c. $\frac{-25}{y^3}$
d. $\frac{6x^2}{y^3}$
e. $\frac{25}{y^3}$
f. None of these.

First find $\frac{dy}{dx}$, we have

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25) \Rightarrow \cancel{2x+2y\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dx}{dy} = ---}$

$2x + 2y\cdot\frac{dy}{dx} = 0$ (*) $\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

Then, using (*) to do "$\frac{d}{dx}$" again, we have

$\frac{d}{dx}\left(2x + 2y\frac{dy}{dx}\right) = 0 \Rightarrow 2 + 2\frac{dy}{dx}\cdot\frac{dy}{dx} + 2y\frac{d^2y}{dx^2} = 0$

product rule

plug in $\frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow 2 + 2\left(-\frac{x}{y}\right)\left(-\frac{x}{y}\right) + 2y\frac{d^2y}{dx^2} = 0$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2-\frac{2x^2}{y^2}}{2y} = \frac{-2y^2-2x^2}{2y^3} = \frac{-2(x^2+y^2)}{2y^3}$

$= -\frac{x^2+y^2}{y^3} = -\frac{25}{y^3}$

since $x^2+y^2=25$