$\frac{1}{x} = x^{-1}$ $\quad \sqrt{x} = x^{\frac{1}{2}}, \sqrt[3]{x} = x^{\frac{1}{3}}$

$\frac{1}{x^2} = x^{-2}$

$\vdots$

$\frac{1}{x^n} = x^{-n}$ $\quad$ and $(x^n)' = n x^{n-1}$ $\forall n \in \mathbb{R} \setminus \{0\}$

Math 1431, Section 17699

EMCF 4 (10 points)

Due 2/14 at 11:59pm

Sol.

Instructions:

- Submit this assignment at http://www.casa.uh.edu under "EMCF" and choose EMCF 4.

A

1. The instantaneous rate of change of y with respect to x at $x = 3$, where $y = x^2 + x - 1$ is

a. 7

b. 8

c. 9

d. 10

e. 13

f. None of these.

means $y'(x)$ at $x=3$

$\Rightarrow y'(x) = 2x+1$, then $y'(3) = 7$

D

2. $\frac{d^2}{dx^2}\left(x^2 - \frac{1}{x}\right) =$

a. $2x - 1$

b. 2

c. $2x + \frac{1}{x^2}$

d. $2 - \frac{2}{x^3}$

e. $2 + \frac{2}{x^3}$

f. None of these.

$\frac{d}{dx}\left(\frac{d}{dx}\left(x^2 - \frac{1}{x}\right)\right) = \frac{d}{dx}\left(2x - (-x^{-2})\right)$

$= \frac{d}{dx}\left(2x + x^{-2}\right) = 2 - 2x^{-3} = 2 - \frac{2}{x^3}$

B

3. Find the value of $\lim_{x\to 0} \frac{\tan(3x)}{x}$.

a. 0

b. 3

c. 9

d. 1/3

e. 1/9

f. None of these.

$= \lim_{x\to 0} \frac{\sin(3x)}{\cos(3x)} \cdot \frac{1}{x} \cdot \frac{3}{3}$

$= \lim_{x\to 0} \frac{\sin(3x)}{3x} \cdot \frac{3}{\cos(3x)}$

$= 1 \cdot \frac{3}{1} = 3$

Slope $= f'(x) = 3x^2 + 3$

@ $x=3$, $f'(3) = 3\cdot 9 + 3 = 30$

B

4. The slope of the tangent line to the curve $f(x) = x^3 + 3x + 3$ at $x = 3$ is 33.

a. True

b. False

C

5. Let $f(x) = \sqrt{x+3}$. Give $f'(1)$. Use the definition of the derivative at a point.

a. 1/2

b. −1/2

c. 1/4

d. −1/4

e. DNE

f. None of these.

$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{h+1+3} - \sqrt{4}}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{h+4} - \sqrt{4})(\sqrt{h+4} + \sqrt{4})}{h\,(\sqrt{h+4} + \sqrt{4})} = \lim_{h\to 0} \frac{1}{\sqrt{h+4} + \sqrt{4}} = \frac{1}{4}$

D

6. At $x = 3$, is the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ continuous and/or differentiable?

a. Continuous only

b. Differentiable only

c. Neither continuous nor differentiable

d. Continuous and differentiable

e. None of these.

I. Continuity ① $\lim_{x\to 3^+} f(x) = 6\cdot 3 - 9 = 9$, ③ $f(3) = 9$

② $\lim_{x\to 3^-} f(x) = 3^2 = 9$, ① = ② = ③ ✓

II. Differentiability ⓐ $\lim_{h\to 0^+} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h\to 0^+} \frac{6(3+h) - 9 - 9}{h}$

$= 6$ and

ⓑ $\lim_{h\to 0^-} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h\to 0^-} \frac{(3+h)^2 - 9}{h}$

$= \lim_{h\to 0} \frac{6h + h^2}{h} = 6$

ⓐ = ⓑ ✓

D

7. Let $g(x) = x^2 f(x)$. Find $g'(3)$, given that $f(3) = 6$ and $f'(3) = 2$.

a. 12

b. 18

c. 36

d. 54

e. 72

f. None of these.

By Product Rule,

$g'(x) = 2x f(x) + x^2 f'(x)$

so $g'(3) = 2\cdot 3 \cdot f(3) + 3^2 f'(3)$

$= 6\cdot 6 + 9\cdot 2$

$= 36 + 18$

$= 54$

E

8. Give the slope of the normal line to the graph of $f(x) = 3x^3 - 4x^2 + 2x - 1$ at the point $x = -1$.

a. $-1/15$
b. $-1/16$
c. $-1/17$
d. $-1/18$
e. $-1/19$
f. None of these.

$f'(x) = $ Slope of $f(x)$ = Slope tangent line $= 9x^2 - 8x + 2$.

$f'(1) = 9(-1)^2 - 8(-1) + 2 = 19$

(Slope of normal) $\cdot$ (Slope of tangent) $= -1$ $\Rightarrow$ Slope of normal $= -\frac{1}{19}$

D

9. Find $f''(x)$ (the second derivative) if $f(x) = \frac{x^2 - 3x}{x^2}$. $= \frac{x^2}{x^2} - \frac{3x}{x^2} = 1 - \frac{3}{x} = 1 - 3 \cdot x^{-1}$

a. $-\frac{9}{x^2}$
b. $\frac{3}{x^2}$
c. $\frac{4x-9}{x^4}$
d. $-\frac{6}{x^3}$
e. None of these.

$f'(x) = 3x^{-2}$

$f''(x) = -6x^{-3} = \frac{-6}{x^3}$

A

10. Give the derivative of $f(x) = 4\sqrt{x} - x^4$.

a. $f'(x) = \frac{2}{\sqrt{x}} - 4x^3$
b. $f'(x) = \frac{1}{\sqrt{x}} - 4x^3$
c. $f'(x) = \frac{2}{\sqrt{x}} - 4x^2$
d. $f'(x) = \frac{1}{\sqrt{x}} - 4x^2$
e. $f'(x) = 2 - 4x$
f. None of these.

$f(x) = 4 \cdot x^{\frac{1}{2}} - x^4$

$f'(x) = 4 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 4x^3$

$= 2x^{-\frac{1}{2}} - 4x^3$

$= \frac{2}{\sqrt{x}} - 4x^3$