Class work 8.

1. $y=\arctan(x^2) \Rightarrow y'=\frac{2x}{1+x^4}$

2. $f(t)=(\ln(7t^5))^3 \Rightarrow f'(t)=3(\ln(7t^5))^2\cdot\frac{35t^4}{7t^5}$

3. $y=e^{\sinh(x)} \Rightarrow y'=\cosh(x)\,e^{\sinh(x)}$

4. $y=\ln(2x^2+\sin x) \Rightarrow y'=\frac{4x+\cos(x)}{2x^2+\sin(x)}$

5. $f(x)=\ln(\ln x^6)=\ln(6\ln x) \Rightarrow f'(x)=\frac{1}{6\ln x}\cdot\frac{6}{x}$.

6. $f(\theta)=\ln(\sqrt{1-\cos^2 2\theta})=\frac{1}{2}\ln(1-\cos^2 2\theta)$

$$\Rightarrow f'(\theta)=\frac{1}{2}\cdot\frac{-2\cos(2\theta)\cdot(-\sin(2\theta))\cdot 2}{1-\cos^2 2\theta}=\frac{2\cos(2\theta)\sin(2\theta)}{1-\cos^2(2\theta)}$$

7. $y=\frac{x^2(5x^2+4)^{\frac{1}{2}}}{9x^2-2}$,

$$\ln y=\ln\left(\frac{x^2(5x^2+4)^{\frac{1}{2}}}{9x^2-2}\right)=\ln(x^2)+\ln(5x^2+4)^{\frac{1}{2}}-\ln(9x^2-2)$$

$$=2\ln x+\frac{1}{2}\ln(5x^2+4)-\ln(9x^2-2)$$

$\downarrow$ derivative

$$\frac{y'}{y}=\frac{2}{x}+\frac{1}{2}\cdot\frac{10x}{5x^2+4}-\frac{18x}{9x^2-2}$$

$$\Rightarrow y'=\left[\frac{2}{x}+\frac{5x}{5x^2+4}-\frac{18x}{9x^2-2}\right]\frac{x^2(5x^2+4)^{\frac{1}{2}}}{9x^2-2}$$

8. $y=\ln|6x^3-5x+1|$. $y'=\frac{18x^2-5}{6x^3-5x+1}$

9. let $f(x) = \int_{-2}^{2x^3} \sqrt{5t^2-3}\,dt$

$\Rightarrow f'(x) = (2x^3)' \cdot \sqrt{5\underbrace{(2x^3)^2}_{4x^6}-3} = 6x^2 \cdot \sqrt{20x^6-3}$

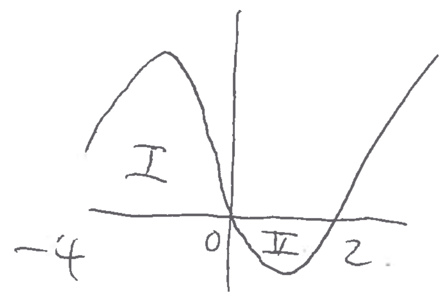
10. let $f(x) = \int_{-3}^{\csc x} \sqrt[3]{(3t^2+1)^2}\,dt$

$\Rightarrow f'(x) = (\csc(x))' \cdot \sqrt[3]{(3\csc^2(x)+1)^2}$

$= -\csc(x)\cot(x) \cdot \sqrt[3]{(3\csc^2(x)+1)^2}$

11. a. Let $F(x) = \int_{-4}^{x} f(t)\,dt$.

$F'(x) = f(x) \Rightarrow F'(1) = f(1) = -1.$



b. $P = \{-4, -3, -2, -1, 0, 1\}$.

$L_f = 1 \cdot 2.8 + 1 \cdot 3 + 1 \cdot 2.5 + 1 \cdot 1.5 + 1 \cdot 0 = 9.8$ (

c. $\frac{d}{dx}\int_{-4}^{x} f(t)\,dt = f(x)$, $f(-2) = 2.5$.

d. $F(0) = \int_{-4}^{0} f(t)\,dt = \mathrm{I} > 0$.

$F(2) = \int_{-4}^{2} f(t)\,dt = \mathrm{I} - \mathrm{II}$ $(\mathrm{II} > 0) \Rightarrow F(0) > F(2)$.

e. F is increasing $\Leftrightarrow F' > 0$ since $F'(x) = f(x)$

$\Rightarrow f(x) > 0 \Rightarrow x \in (-4, 0) \cup (2.2, 3)$

f. $F(-2) = \int_{-4}^{-2} f(t)\,dt =$

$F(2) = \int_{-4}^{2} f(t)\,dt =$

12. Given $\int_2^5 f(x)dx = 5$, $\int_4^5 f(x)dx = 2$.

a. $\int_5^5 f(x)dx = 0$.

b. $\int_5^4 f(x)dx = -\int_4^5 f(x)dx = -2$

c. $\int_2^4 f(x)dx = \int_2^5 f(x)dx - \int_4^5 f(x)dx = 5-2=3$