Group Members: __Sol z,__ _____

Classwork 7 – Optimization

With your group, set up and solve the following optimization problem. Show all work at each step.

A 400-room hotel in Las Vegas is filled to capacity every night at $75.00 a room. For each $1.00 increase in price, 4 fewer rooms are booked. If each occupied room costs $15.00 to service per day, how much should the management charge for each room to maximize profit?

a) Find a formula for the function you want to maximize.

Let x be the increasing in price.

Then the profit for each room is $75 + x - 15 = 60 + x$, and the number of room rented is $400 - 4x$

Thus, Tatal profit will be "the profit for each room times the number of room"

$$f(x) = (60 + x)(400 - 4x)$$
$$= -4x^2 + 160x + 24000$$

b) What is a feasible domain for the function in part (a)?

The number of room should be between 0 and 400

$\Rightarrow \quad 0 \le 400 - 4x \le 400 \quad \Rightarrow \quad -400 \le -4x \le 0$

$\Rightarrow \quad 0 \le 4x \le 400 \quad \Rightarrow \quad 0 \le x \le 100.$

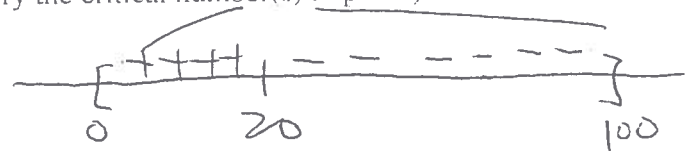c) Determine the critical number(s) of the function.

$$f(x) = -4x^2 + 160x + 24000$$

$$f'(x) = -8x + 160 = 0$$

$$x = 20.$$

d) Use the first or second derivative test to classify the critical number(s) in part c).

First derivative test,

$f(20)$ is a local max $\Rightarrow$ max profit,

Second derivative test

$$f''(x) = -8 < 0 \Rightarrow f(20) \text{ is a local max.}$$

e) At what price does the hotel maximize profit? What is the profit at this price?

$$x = 20$$

$$f(20) = -4(20)^2 + 160 \cdot 20 + 24000$$

$$= 1600 + 24000 = 25600.$$