

Group Members: _____

Sol. I.

Classwork 7 – Optimization

With your group, set up and solve the following optimization problem. Show all work at each step.

A 400-room hotel in Las Vegas is filled to capacity every night at \$75.00 a room. For each \$1.00 increase in price, 4 fewer rooms are booked. If each occupied room costs \$15.00 to service per day, how much should the management charge for each room to maximize profit?

a) Find a formula for the function you want to maximize.

Let rent for each room be R dollar,

Then the profit for each room is $R - 15$, and
the number of room rented is $400 - 4(R - 75)$
 $= 700 - 4R$

Thus, total profit will be

$$f(R) = (R - 15)(700 - 4R)$$
$$= -4R^2 + 760R - 10500$$

b) What is a feasible domain for the function in part (a)?

The number of Room should be between 0 and 400.

$$0 \leq 700 - 4R \leq 400$$

$$\Rightarrow 300 \leq 4R \leq 700 \Rightarrow \frac{300}{4} \leq R \leq \frac{700}{4}$$

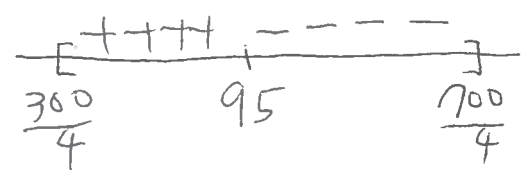
c) Determine the critical number(s) of the function.

$$f(R) = -4R^2 + 760R - 10500$$

$$f'(R) = -8R + 760 = 0$$

$$R = \frac{760}{8} = 95$$

d) Use the first or second derivative test to classify the critical number(s) in part c).

First:  $\Rightarrow f(95)$ is a local max.

Second: $f''(R) = -8 < 0 \Rightarrow f(95)$ is a local max.

e) At what price does the hotel maximize profit? What is the profit at this price?

$$R = 95$$

$$\text{Profit: } f(95) = 25600$$