Group Memebers: Sol

Classwork 6 – Inverse Functions

1. Define one-to-one function.

$f(x_1)=f(x_2)$ implies $x_1=x_2$.

2. Is $f(x)=x^2-3x+2$ one-to-one?

$f'(x)=2x-3 \Rightarrow$ NOT monotone $\Rightarrow f$ is NOT one-to-one

3. Is $f(x)=(x-2)^{3/2}+1$ one-to-one? $(D(f)=\{x\geq 2\})$

$f'(x)=\frac{3}{2}(x-2)^{\frac{1}{2}}\geq 0$ as $x\geq 2 \Rightarrow f$ is always increasing on $D(f)$

$\Rightarrow f$ is one-to-one.

4. Is $f(x)=(x-2)^{2/3}+1$ one-to-one? $(D(f)=\mathbb{R})$

$f'(x)=\frac{2}{3}(x-2)^{-\frac{1}{3}}=\frac{2}{3}\frac{1}{(x-2)^{\frac{1}{3}}} \Rightarrow$ NOT monotone $\Rightarrow f$ is NOT one-to-one

If a function is one-to-one, then it has an inverse. (Remember, domain of *f* equals the range of f^{-1})

5. Determine if $f(x)=4x^5+1$ is one-to-one and if so, find $f^{-1}(x)$.

$f'(x)=20x^4\geq 0 \Rightarrow f$ is always Increasing $\Rightarrow f$ is invertible.

Find f^{-1}: ① Let $y=f(x)=4x^5+1$ ② Switch x and $y \Rightarrow x=4y^5+1$

③ Solve y: $x-1=4y^5 \Rightarrow \frac{x-1}{4}=y^5 \Rightarrow y=\sqrt[5]{\frac{x-1}{4}}$

6. Determine if $f(x)=x^{9/7}$ is one-to-one and if so, find $f^{-1}(x)$.

$f'(x)=\frac{9}{7}x^{\frac{2}{7}}\geq 0 \Rightarrow f$ is always Increasing $\Rightarrow f$ is invertible

Find f^{-1}! ① Let $y=f(x)=x^{\frac{9}{7}}$ ② switch x and y: $x=y^{\frac{9}{7}}$

③ solve y: $y=x^{\frac{7}{9}}$

Derivative of Inverse: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

7. Suppose f has an inverse and $f(2)=5$. $f'(2)=3/7$. Find $(f^{-1})'(5)$

$f(2)=5 \Rightarrow f^{-1}(5)=2$

$$(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{\frac{3}{7}} = \frac{7}{3}$$

8. $f(x)=x^3+2$, $f(3)=29$, find $(f^{-1})'(29)$

$f(3)=29 \Rightarrow f^{-1}(29)=3$

$f'(x)=3x^2$

$$\Rightarrow (f^{-1})'(29) = \frac{1}{f'(3)} = \frac{1}{3\cdot 3^2} = \frac{1}{27}$$

9. $f(x)$ passes through the points (3 , -2) and (-2 , 5). The slope of the tangent line to the graph of $f(x)$ at x = -2 is -1/4. Evaluate the derivative of the inverse of f at 5.

f passes $(3,-2) \Rightarrow f(3)=-2$

f passes $(-2,5) \Rightarrow f(-2)=5 \Rightarrow f^{-1}(5)=-2$

The slope of tangent line of f at $x=-2$ is $-\frac{1}{4} \Rightarrow f'(-2)=-\frac{1}{4}$.

$$(f^{-1})'(5) = \frac{1}{f'(-2)} = \frac{1}{-\frac{1}{4}} = -4$$

10. Suppose that f has an inverse and $f(-20) = -2$, $f'(-20) = 4/3$. If $g = \dfrac{1}{f^{-1}}$, what is $g'(-2)$? Hint: use the reciprocal rule to find g' first.

$$f(-20)=-2 \Rightarrow (f^{-1})(-2) = -20 \Rightarrow (f^{-1})'(-2) = \frac{1}{f'(20)} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

quotient rule

$$g'(-2) = \left(\frac{1}{f^{-1}}\right)'(-2) \overset{\downarrow}{=} \left(\frac{-(f^{-1})'}{(f^{-1})^2}\right)(-2) = \frac{-(f^{-1})'(-2)}{(f^{-1})^2(-2)}$$

$$= \frac{-\frac{3}{4}}{(-20)^2} = -\frac{3}{1600}$$