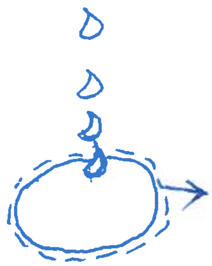
Group Memebers: Sol

Classwork 3 - Related Rates

1. Water leaking onto a floor creates a circular pool with an area that increases at the rate of 3 cm^2/min. How fast is the radius of the pool increasing when the radius is 10 cm?

Picture:



let A be area of circular pool.
r be the radius of this pool.

Know: $A = \pi r^2$ and $\frac{dA}{dt} = 3\left(\frac{cm^2}{min}\right)$.

Find: $\frac{dr}{dt}$ as $r = 10cm$.

Equation: $\frac{d(\pi r^2)}{dt} = 3$.

Work: $\Rightarrow \frac{d(\pi r^2)}{dt} = \pi 2r \cdot \frac{dr}{dt} = 3$.

$\Rightarrow \frac{dr}{dt} = \frac{3}{2\pi r}$

As $r = 10cm$, we have $\left.\frac{dr}{dt}\right|_{r=10} = \frac{3}{2\pi \cdot 10} = \frac{3}{20\pi}$

Final Answer: ______

Group Memebers: ______________________ ______________________

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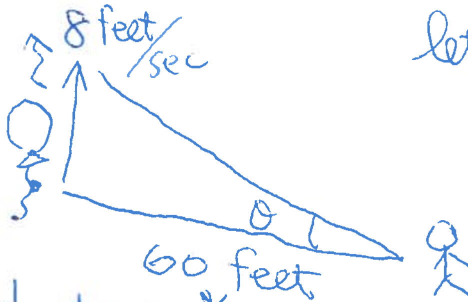
Classwork 3 - Related Rates

2. A balloon rises at the rate of 8 ft./sec. from a point on the ground 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.

Picture:



let the height of ballon be x feet.

Know: $\frac{dx}{dt}=8$ and $\tan\theta=\frac{x}{60}$

Find: $\left.\frac{d\theta}{dt}\right|_{as\ x=25}$

Equation: $\frac{d}{dt}\left(\tan\theta=\frac{x}{60}\right)$

Work: $\frac{d}{dt}(\tan\theta)=\frac{d}{dt}\left(\frac{x}{60}\right).$

$\Rightarrow \sec^2\theta\cdot\frac{d\theta}{dt}=\frac{1}{60}\frac{dx}{dt}$

$\Rightarrow \frac{d\theta}{dt}=\frac{1}{60}\cdot\frac{dx}{dt}\cdot\frac{1}{\sec^2\theta}$

$=\frac{8}{60}\cdot\frac{1}{\sec^2\theta}$

As $x=25$, we have $\tan\theta=\frac{25}{60}\Rightarrow\sec^2\theta=1+\tan^2\theta=\frac{625}{3600}+1$

$=\frac{4225}{3600}$

Final Answer: Then $\left.\frac{d\theta}{dt}\right|_{x=25}=\frac{8}{\cancel{60}}\cdot\frac{\cancel{3600}^{60}}{4225}=\frac{480}{4225}$

Group Memebers: ______________________ ______________________

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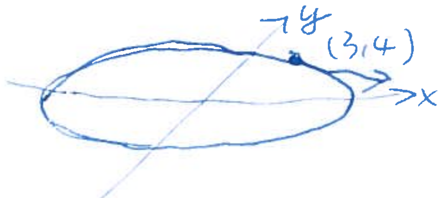
Classwork 3 - Related Rates

3. A particle is moving in a circular orbit $x^2 + y^2 = 25$. As it passes through the point (3, 4), its y coordinate is decreasing at the rate of 2 units per second. How is the x coordinate changing?

Picture:



Know: $\left.\frac{dy}{dt}\right|_{(3,4)} = -2 \frac{\text{units}}{\text{second}}$

Find: $\frac{dx}{dt}$.

Equation: $\frac{d}{dt}(x^2+y^2=25)$.

Work: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \frac{dx}{dt} = \left(-2y\frac{dy}{dt}\right)\cdot\frac{1}{2x}$

$\Rightarrow \left.\frac{dx}{dt}\right|_{(3,4)} = -2\cdot 4\cdot -2\cdot\frac{1}{2\cdot 3}$

$= \frac{8}{3} \frac{\text{units}}{\text{second}}$

Final Answer: ______________________

Group Memebers: ______________________ ______________________

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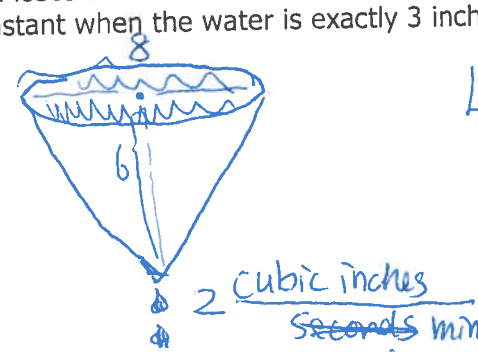
Classwork 3 - Related Rates

4. A conical paper cup 8 inches across the top and 6 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep?

Picture:



Let $\underset{\sim}{V}$ be the volume of water. the heigh of water in cup be $\underset{\sim}{h}$. the radius be $\underset{\sim}{r}$

Know: $\frac{dV}{dt} = -2$, $V = \frac{1}{3} h \pi r^2$, $\frac{r}{h} = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow r = \frac{2}{3} h.$

$\Rightarrow V = \frac{1}{3} h \pi \left(\frac{2}{3} h\right)^2$

$= \frac{1}{3} h \pi \cdot \frac{4}{9} h^2$

$= \frac{4}{27} \pi h^3$

Find: $\left.\frac{dh}{dt}\right|_{h=3}$

Equation: $\frac{d}{dt}\left(V = \frac{4}{27} \pi h^3\right)$

Work: $\frac{dV}{dt} = \frac{4}{27} \pi \cdot 3h^2 \frac{dh}{dt}$

$-2 = \frac{4}{27} \pi \cdot 3(3)^2 \left.\frac{dh}{dt}\right|_{h=3}$

$\left.\frac{dh}{dt}\right|_{h=3} = \frac{-2}{4\pi} = -\frac{1}{2\pi}$

Final Answer: ______________________