Math 1450, Honor Calculus Practice 2, Fall 2015.

September 4, 2015

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1. Find a formula for the inverse of the function $f(x) = \dfrac{4x-1}{2x+3}$.

Step 1. Let $y = f(x) \Rightarrow y = \dfrac{4x-1}{2x+3}$

Step 2. Switch x and y $\Rightarrow x = \dfrac{4y-1}{2y+3}$

Step 3. Solve y $\Rightarrow \dfrac{x}{1} = \dfrac{4y-1}{2y+3} \Rightarrow x(2y+3) = 4y-1$

(collect y in one side and collect non-y in the other side)

$\Rightarrow 2xy + 3x = 4y - 1$

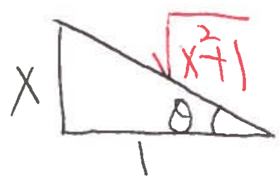
$\Rightarrow 2xy - 4y = -3x - 1$

$\Rightarrow (2x-4)y = -3x - 1$

$\Rightarrow \boxed{y = \dfrac{-3x-1}{2x-4}} = f^{-1}(x)$

2. Simplify the expession $\sin(\tan^{-1}(x))$.

Let $\theta = \tan^{-1}(x) \Rightarrow \dfrac{x}{1} = \tan(\theta)$



Then $\sin(\tan^{-1}(x))$

$= \sin(\theta) = \dfrac{x}{\sqrt{x^2+1}}$

3. Prove that $\lim_{x\to 0} x^4 \cos(\frac{2}{x}) = 0$. (By Squeeze's Thm)

Since $-1 \le \cos(\frac{2}{x}) \le 1$ and $x^4 \ge 0$ $\forall x \in \mathbb{R}$ (for all)

Then $-x^4 \le x^4 \cos(\frac{2}{x}) \le x^4$. Thus, $\lim_{x\to 0} x^4 = 0$, $\lim_{x\to 0} -x^4 = 0$,

and $0 = \lim_{x\to 0} -x^4 \le \lim_{x\to 0} x^4 \cos(\frac{2}{x}) \le \lim_{x\to 0} x^4 = 0 \Rightarrow \lim_{x\to 0} x^4 \cos(\frac{2}{x}) = 0$

4. Find the limit $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$ if it exists. If not, explain why?

By def. of $\dfrac{1}{|x|}$, we have $\dfrac{1}{|x|} = \begin{cases} \frac{1}{x}, & x > 0; \\ -\frac{1}{x}, & x < 0. \end{cases}$

$\Rightarrow \dfrac{1}{x} - \dfrac{1}{|x|} = \begin{cases} 0, & x > 0; \\ \frac{2}{x}, & x < 0 \end{cases} \Rightarrow \lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) = 0.$