

August 26, 2015

PSID:

Name:

Sol

1. $f(x) = x - 2, g(x) = x^2 + 3x + 4$. Find $g \circ f$ and its domain.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = (x-2)^2 + 3(x-2) + 4 \\&= x^2 - 4x + 4 + 3x - 6 + 4 \\&= x^2 - x + 2.\end{aligned}$$

Domain of $g \circ f$: $D(g \circ f) = \mathbb{R}$ or $(-\infty, \infty)$ or $\{x | -\infty < x < \infty\}$

2. Find the domain of function $f(x) = \sqrt{x} + \sqrt[3]{x}$.

$$D(\sqrt{x}) = [0, \infty), \quad D(\sqrt[3]{x}) = (-\infty, \infty)$$

$$\Rightarrow D(f) = [0, \infty) \cap (-\infty, \infty) = [0, \infty)$$

3. Let $f(x) = x + 4$ and $h(x) = 4x - 1$. Find function g such that $g \circ f = h$.

$$4x - 1 = h(x) = g(f(x)) = \underbrace{4(x+4)}_{4x+16} - 17$$

$$\Rightarrow g(x) = 4x - 17.$$

4. Suppose f and g are odd functions. Are $f + g$ and fg odd? Explain it!

f and g are odd functions $\Rightarrow f(-x) = -f(x), g(-x) = -g(x)$

Then

$$\textcircled{1} (f+g)(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f+g)(x) \Rightarrow \text{odd.}$$

$$\textcircled{2} (fg)(-x) = f(-x)g(-x) = [-f(x)] \cdot [-g(x)] = f(x)g(x) = (fg)(x) \Rightarrow \text{even.}$$