Honor Calculus, Math 1450 - Assignment 7 - Solution

Q1.

- $3+4i$

  $|3+4i| = \sqrt{3^2+4^2} = 5$

  Argument: $\tan\theta = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$.

- $(3+4i)^{-1} = \dfrac{1}{5e^{i\theta}} = \dfrac{1}{5}e^{-i\theta} \Rightarrow |(3+4i)^{-1}| = \dfrac{1}{5}$ and

  argument will be $-\theta$ where $\tan\theta = \dfrac{3}{4}$.

sin +	All +
tan +	cos +

- $(1-i)^5$.

  since $1-i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \sqrt{2}\, e^{\frac{7}{4}\pi i}$

  Then $(1-i)^5 = \left(\sqrt{2}\, e^{\frac{7}{4}\pi i}\right)^5 = 4\sqrt{2} \cdot e^{\frac{35}{4}\pi i} = 4\sqrt{2}\, e^{\frac{3\pi}{4} i}$

  $|(1-i)^5| = 4\sqrt{2}$, argument: $\frac{3\pi}{4}$.

- $2+3i$

  $|2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$.

  Argument: θ where $\tan\theta = \dfrac{3}{2}$

Q2.

- $x^2+4=0 \Rightarrow x^2 = -4 \Rightarrow x = \pm 2i$.

Q2. • $x^2+x+1=0$

$$\Rightarrow x=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm\sqrt{3}i}{2}.$$

• $X^4+X^2+1=0$

$$\Rightarrow X^2=\frac{-1\pm\sqrt{3}}{2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i=e^{\frac{2}{3}\pi i} \text{ or } e^{\frac{4}{3}\pi i}$$

$$X=\pm e^{\frac{\pi}{3}i},\ \pm e^{\frac{2\pi}{3}i}$$

Q3

• $\bar{z}=-z$ $\quad a,b\in\mathbb{R}.$

let $z=a+ib$, $\wedge$ Then $\begin{array}{l}\bar{z}=a-ib\\ -z=-a-ib\end{array}\Rightarrow$

$$a-ib=-a-ib\Rightarrow a=-a\Rightarrow a=0.$$

$\Rightarrow z=ib.\quad b\in\mathbb{R}.$

• $|z|<1.$

let $z=re^{i\theta}$, $r\in\mathbb{R}.\ 0\leq\theta<2\pi.$

Then $|z|=|re^{i\theta}|\underset{\uparrow}{=}|r|<1\Rightarrow|r|<1.$

$|e^{i\theta}|=1$

• $|z-(1+i)|<4$

let $z=re^{i\theta}$, then $|z-(1+i)|=|re^{i\theta}-\sqrt{2}e^{\frac{\pi}{4}i}|=|r-\sqrt{2}|<4.$

$\Rightarrow|r-\sqrt{2}|<4.$

Q4

let $z = a+ib$, $a,b \in \mathbb{R}$.

Then $\frac{z+\bar{z}}{2} = \frac{a+ib+(a-ib)}{2} = a$ which is real part of z.

and $\frac{z-\bar{z}}{2i} = \frac{a+ib-(a-ib)}{2i} = b$ which is imaginary part of z.

Q4. $z^3 = 1 = e^{i(2k\pi)}$, $k \in \mathbb{N}$.

Then $z = e^{i\left(\frac{2k\pi}{3}\right)}$ $k \in \mathbb{N}$.

$\Rightarrow$ $z = e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}, e^{i\frac{6\pi}{3}}, \ldots$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}-\frac{\sqrt{3}}{2}i, 1.$

Q5. $z^n - 1 = 0 \Rightarrow (z-1)(z^{n-1}+z^{n-2}+\cdots+z^2+z+1) = 0$

Since $z-1 \neq 0 \Rightarrow (z^{n-1}+z^{n-2}+\cdots+z^2+z+1) = 0$.