Honors Calculus, Math 1450- HW 4 (due Tuesday 6th October)

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All section references are to Stewart 6th edition. Show all working and write your answers neatly. Staple your work.

(1) (Mean value theorem)

(a) Show that if $f(x)$ is differentiable on $\mathbb{R}$ and has two roots then $f^{'}(x)$ has at least one root.

(b) Show that if $f(x)$ is twice differentiable on $\mathbb{R}$ and has three roots. Show that $f^{''}(x)$ has at least one root.

(2) Use the Mean Value Theorem to show that if $f^{'}(x) > g^{'}(x)$ on (a, b) and $f(a) = f(b)$ then $f(x) > g(x)$ for all $x \in (a, b)$.

(3) Use (2) (or another argument) to show that $\sqrt{x+1} < 1 + \frac{x}{2}$ if $x > 0$.

(4) Section 4.3 (Shape of a graph): 14, 16, 18, 26, 68, 72, 76

(5) Find the dimensions of the circular cylinder of volume 1 which has the least surface area (counting the areas of the faces at the top and bottom).

(6) A projectile is fired from the ground with initial velocity v_0 at an angle θ so that it has a vertical component of velocity $v_0 \sin\theta$ and a horizontal component $v_0 \cos\theta$. From Newton's laws of gravitation we know that its height above the ground satisfies $y(t) = -16t^2 + (v_0 \sin\theta)t$ while its horizontal velocity is constant at $v_0 \cos\theta$ (we neglect air resistance).
(a) Show that the path of the projectile is a parabola.
(b) Find the angle θ which will maximize the range i.e. the horizontal distance traveled by the projectile before hitting the ground.

(7) (a) Show that if a particle of mass m moves on the x-axis so that its position is $x(t)$, velocity $\dot{x}(t)$ and acceleration $\ddot{x}(t)$ satisfy $m\ddot{x} = -\frac{dV}{dx}$ for some function $V(x)$ then $V(x) + \frac{1}{2}m\dot{x}^2$ remains constant. In physics V would be called a potential energy.
(b) The equation for the motion of a spring is often given as $m\ddot{x} = -kx$ where $k > 0$ is called the stiffness of the spring. Suppose a particle undergoes motion described by $m\ddot{x} = -kx$ where $m = 1$, $k = 2$ and $\dot{x}(0) = 3$. Using (a) find the maximum distance of the particle from the origin.

(8) Section 4.4 (limits): 8, 10, 20, 28, 40, 42, 56