Honors Calculus, Math 1450- HW 2 (due Thursday 9th September)

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(1) Suppose that a sphere has radius $r(t)$ and $\frac{dr}{dt} = r^{1/3}$. Find the rate of change with respect to time when $r = 2$ of the : (a) volume of the sphere; (b) surface area of the sphere.

(2) Show that $\frac{d}{dx} \csc x = -\csc x \cot x$

(3) Suppose, instead of measuring an angle θ in radians, we measure θ in degrees, where 2π radians equals 360 degrees. Show that

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \frac{\pi}{180}$$

(4) Find $\frac{dw}{dt}$ if

(i) $w = \tan x$ and $x = 2t^2 + 1$

(ii) $w = 2^x$ and $x = \sin(\sqrt{t})$

(5) Suppose $f(x)$ is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable with $f^{'}(x) \neq 0$ for any x. Show that

$$(f^{-1})^{'}(a) = \frac{1}{f'(f^{-1}(a))}$$

(6) The number a is called a double root of the polynomial function f if $f(x) = (x-a)^2 g(x)$ for some polynomial function g. Show that a is a double root of f if and only if a is a root of both $f^{'}(x)$ and $f(x)$.

(6) Find $\lim_{\theta \to 0} \frac{\sin(7\theta)}{\sin(2\theta)}$

(7) Differentiate i.e. find $f'(x)$ where

(i) $f(x) = \ln(\ln(x^2 + 1))$

(ii) $f(x) = e^{x^2 \tan(x)}$

(iii) $f(x) = x^{x^2}$

(8) Show that $f(x) = x^{3/2}$ is differentiable at 0 but not twice differentiable at $x = 0$.

(9) Differentiate

(i) $f(x) = \ln(\ln(x^4 + 1))$

(ii) $f(x) = e^{x^2 \sin(x)}$

(iii) $f(x) = \cot^2(x)$

(10) Differentiate

(i) $f(x) = \ln(\sec x + \tan x)$

(ii) $g(t) = e^{\frac{t}{t^2+6}}$

(11) Find $\frac{d^2y}{dx^2}$ if x and y are related by the equation

(i) $x^3 + y^3 = 1$

(ii) $y + \sin(y) = x$

(12) Find the equation of the tangent line to $x^2y - 5xy^2 = -6$ at the point $(3, 1)$.

(13) Section 3.5: Question 58.

(14) Use logarithmic differentiation to find the derivative of $y = (2x+1)^3(x^4-2)^5$.

(15) The definition of $|x|$ is

$$|x| = \begin{cases} x & \text{if } x \geq 0; \\ -x & \text{if } x < 0. \end{cases}$$

Show that $f'(x) = \frac{1}{x}$ if $x \neq 0$ and $f(x) = \ln(|x|)$.