

Honors Calculus, Math 1450- Assignment 1

Dr Matthew Nicol, PGH 651A

Due Thursday 9/3/2015 2.30pm

(1) Differentiate

$$f(x) = x^{-2}$$

$$f(x) = x^\pi$$

(2) From the definition of the derivative of $f(x)$ at $x = c$ as

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

find $f'(-2)$ for $f(x) = x^3$.

(3) Find

$$(fg)'(1)$$

if $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = -3$.

(4) Find the following derivatives $\frac{df}{dx}$ when

(a) $f(x) = 2x^3$

(b) $f(x) = \frac{1}{x^2+1}$

(c) $f(x) = \frac{2x^3}{x+1}$

At which point is the derivative in (c) not defined?

(5) Express the derivative $\frac{d}{dx}[(f(x))^2 + 1]$ in terms of f' and f . Show that if $|f(0)| < 2$ and $|f'(0)| < 1$ then $\frac{d}{dx}[(f(x))^2 + 1] < 4$ at $x = 0$.

(6) Suppose $g(x) = f(cx)$. Show from the definition that

$$g'(x) = cf'(cx)$$

(7) Find $f''(x)$ if

(a) $f(x) = x^3 + 3x^2$

(b) $f'(x) = x^3$

(c) $f(x) = ax^2 + bx + c$ where a, b, c are constants.

(8) Find the equation of the tangent line to the curve $y = \frac{8}{x^2+x+2}$ at $x = 2$.

(9) Suppose f is differentiable. Find

(a)
$$\lim_{h \rightarrow 0} \frac{f(x+5h) - f(x)}{h}$$

and

(b)
$$\lim_{h \rightarrow 0} \frac{f(x) - f(x+h)}{h}$$

in terms of the function f' .

(10) Find the equation of the normal to the curve $y = e^x$ at $x = (2, e^2)$.

(11) Find $\frac{dy}{dx}$ where

$$y = (x^{-1} + 2x)^5$$

(12) Find $\frac{dy}{dx}$ where

$$xy + yx^2 = x + y$$

and hence evaluate $\frac{dy}{dx}$ at the point $(1, 1)$.

(13) Find equations of the tangent lines to the curve

$$y = \frac{1-x}{x+1}$$

that are parallel to $3x + 2y = 1$.

(14) Differentiate

$$x^2 \tan^{-1}(2x)$$

(15) Find y' where

$$y = \tan^2(\sin \theta)$$

(16) Find $\frac{dy}{dx}$ where

$$y = \cos^2(4x) + \sin^2(2x)$$

and hence find the equation of the tangent line at $x = \frac{\pi}{4}$.

(17) Show that $f'(x) \leq f(x)$ if

$$f(x) = e^{\sin x}$$

(18) A particle moves along a horizontal line so that its coordinate at time t is

$$x(t) = \sqrt{1 + 4t^2}$$

for $t \geq 0$. Find the velocity and the acceleration as functions of time. What is the limiting velocity i.e. is there a limit to the velocity as $t \rightarrow \infty$?

(19) A particle is constrained to move along an ellipse whose equation is $\frac{y^2}{4} + x^2 = 1$. In this case the particles x and y coordinates are functions of time, $x(t)$ and $y(t)$ satisfying the equation of the ellipse. Find the point or points on the ellipse at which the velocity in the vertical direction equals the velocity in the horizontal direction.

(20) Show that $y = \sin(t)$ and $y = \cos(t)$ satisfy the differential equation

$$\frac{d^2y}{dt^2} = -y$$

Find a solution to the differential equation

$$\frac{d^2y}{dt^2} = -4y$$