Honors Calculus, Midterm1.

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ATTEMPT ALL QUESTIONS. SHOW ALL WORKING. POINTS WILL NOT BE AWARDED IF WORKING IS NOT SHOWN. NO PROGRAMMABLE CALCULATORS ARE TO BE USED. TIME ALLOWED: 80 MINUTES

Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

GOOD LUCK!

(1) [10 points] Find the absolute maximum, absolute minimum and all critical points of

$$f(x) = x\sqrt{9 - x^2}$$

on the closed interval $[-3, 3]$.

(2)[14 points] (a) Suppose $x \geq 0$, $y \geq 0$ and $x + y = 1$. Find the maximum value of

$$-x \ln(x) - y \ln(y)$$

(b) Find the linearization of $\sin x$ near $a = \frac{\pi}{4}$ and use it to estimate $\sin(\frac{5\pi}{4})$.

(3) [10 points] A particle with coordinates $(x(t), y(t))$ is constrained to move on the ellipse

$$\frac{y^2}{4} + x^2 = 1$$

Find the points on the ellipse where $\frac{dx}{dt} = \frac{dy}{dt}$. *Hint: This is a related rates problem.*

(4) [16 points] (a) State without proof the following limits:

(i) $\lim_{x\to 0}(\frac{\sin 3x}{x})^4$

(ii) $\lim_{x\to\infty}\frac{x^2+\sqrt{x}}{x^2+x^{3/2}+1}$

(iii) $\lim_{x\to 0} e^x \sin(x)$

(iv) $\lim_{x\to\infty} e^{-\sqrt{x}}x^2$

(b) Draw the graph of a differentiable function on $[0,\infty)$ which has all the following properties:

- f has a global minimum at $x=0$ and $f(0)=0$
- f is concave up on $[0,1)$ with a point of inflection at $x=1$
- f has a global maximum at $x=2$ and $f(2)=4$
- f is increasing on $[0,2)$ and decreasing on $(2,\infty)$
- $\lim_{x\to\infty} f(x)=1$

(5) [10 points] The rate of radioactive decay of cesium of mass P with respect to time is proportional to the mass of cesium remaining.

(a) Write down an equation expressing the statement above.

(b) Suppose that 200 grams of cesium has decayed to 50 grams over the period of 4 months, at which time another 10 grams is added. Find the total amount of the 60 grams of Cesium remaining 5 months after the 10 grams was added.