Honors Calculus, Sample Final 1.

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ATTEMPT ALL QUESTIONS. SHOW ALL WORKING. POINTS WILL NOT BE AWARDED IF WORKING IS NOT SHOWN. NO PROGRAMMABLE CALCULATORS ARE TO BE USED. TIME ALLOWED: 80 MINUTES

Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

GOOD LUCK!

(1) [10 points] (a) What does it mean for a series

$$\sum_{n=1}^{\infty} a_n$$

to converge or diverge?

(b) Explain the difference between a conditionally convergent and an absolutely convergent series. In your discussion you should give an example of a conditionally convergent series, justifying why it is conditionally convergent.

(2)[20 points] Test the following series for convergence, stating carefully your reasoning.

$$(a) \sum_{n=1}^{\infty} \frac{n^2 + \sqrt{n} + 1}{2^n}$$

$$(b) \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln n}$$

Hint: Recall $\lim_{n \to \infty} \frac{\ln n}{n^p} = 0$ *for any* $p > 0$.

$$(c) \sum_{n=2}^{\infty} \frac{1}{n^{\sqrt{2}} \ln(n)}$$

$$(d) \sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$(e) \sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(3) [10 points] (i) Find the power series expansion of

$$\frac{1}{1+3x^3}$$

for small values of x near $a = 0$. What is the radius of convergence? For what values of x does the power series converge?

(ii) Define

$$f(x) = \frac{1}{1+3x^3}$$

and using (i) or otherwise find $f^{(6)}(0)$.

(4) [10 points] Find the 3rd order Taylor polynomial of the following function about the indicated point.

$$f(x) = \frac{1}{\sqrt{x}}, \quad a = 9$$

(5) (a) [4 points] Describe the behavior of a function $f(x)$ defined by a power series

$$f(x) = \sum_{n=1}^{\infty} a_n (x-a)^n$$

with regard to differentiability and integrability at points $|x - a| < R$ where R is the radius of convergence.

(b)[6 points] Using (a) or otherwise find the Taylor expansion of

$$\tan^{-1}(x)$$

about $a = 0$. *Hint:* $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$.

(6) [10 points] Let $T_n(x)$ be the nth Taylor polynomial for the function $f(x) = \cos(x)$ around $a = 0$. Give an estimate for the nth remainder term $R_n(x) = \cos(x) - T_n(x)$ and show that $\cos(x)$ equals its Taylor series for all real numbers x.