

2014/07/11 Summer Math 1432. "Turn it in Lab 1"

$$\bullet \int \frac{\csc^2(t)}{2+\cot(t)} dt$$

$$\text{let } u = 2 + \cot(t), \quad du = -\csc^2(t) dt$$

$$\Rightarrow \int \frac{\csc^2(t)}{2+\cot(t)} dt \stackrel{\downarrow}{=} \int \frac{-du}{u} = -\ln|u| + C$$

$$= \boxed{-\ln|2+\cot(t)| + C}$$

$$\bullet \int \frac{\cos\theta}{\sin\theta} d\theta \quad (\text{or } \int \cot\theta d\theta) \quad (\text{compare with } \int \tan\theta d\theta)$$

$$\text{let } u = \sin\theta, \quad du = \cos\theta d\theta$$

$$\Rightarrow \int \frac{\cos\theta}{\sin\theta} d\theta \stackrel{\downarrow}{=} \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sin\theta| + C}$$

$$\bullet \int \sec(t) dt \quad (\text{Hint: times } \frac{\sec(t)+\tan(t)}{\sec(t)+\tan(t)})$$

$$= \int \sec(t) \cdot \frac{\sec(t)+\tan(t)}{\sec(t)+\tan(t)} dt = \int \frac{\sec^2(t) + \sec(t)\tan(t)}{\sec(t)+\tan(t)} dt$$

$$\left(\text{let } u = \sec(t) + \tan(t), \quad du = [\sec(t)\tan(t) + \sec^2(t)] dt \right)$$

$$\stackrel{\downarrow}{=} \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sec(t) + \tan(t)| + C}$$

$$\circ \frac{d}{dx} e^{-x^2} \quad (\text{Hint: Chain Rule})$$

$$= e^{-x^2} \cdot (-2x) = \boxed{-2x e^{-x^2}}$$

$$\circ \text{ Given } y = x^2 e^{2x} - e^x \ln x \quad (\text{Hint: Product Rule \& Chain Rule})$$

$$\Rightarrow y' = \frac{dy}{dx} = 2x e^{2x} + x^2 \cdot 2e^{2x} - e^x \ln x - \frac{e^x}{x}$$

$$= \boxed{2x e^{2x} + 2x^2 e^{2x} - e^x \ln x - \frac{e^x}{x}}$$

$$\circ \text{ Given } y = \ln \sqrt{e^x + 4x} = \ln (e^x + 4x)^{\frac{1}{2}} = \frac{1}{2} \ln (e^x + 4x)$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{d}{dx} \left[\frac{1}{2} \ln (e^x + 4x) \right] = \frac{1}{2 \cdot (e^x + 4x)} \cdot (e^x + 4)$$

$$= \boxed{\frac{e^x + 4}{2 \cdot (e^x + 4x)}}$$