# Math 1432

## Exam 4 Review

1. In each of the following, determine whether or not L'Hopital's Rule applies. If it applies, state the indeterminate form then find the limit.

   a. $\lim\limits_{x->0} \dfrac{1+x-e^x}{x^2}$

   b. $\lim\limits_{x->1} \dfrac{x+\ln x}{2x^2}$

   c. $\lim\limits_{x->\pi/2} (x-\pi/2)\tan x$

   d. $\lim\limits_{x->\infty} \left(1+\dfrac{2}{x}\right)^{2x}$

   e. $\lim\limits_{x->0} \dfrac{1-\cos x}{x^2}$

   f. $\lim\limits_{x->0+} \left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x}}\right)$

   g. $\lim\limits_{n\to\infty} \dfrac{\ln(n+4)}{n+2}$

   h. $\lim\limits_{n\to\infty} (3n)^{\frac{2}{n}}$

   i. $\lim\limits_{n\to\infty} \left(1+\dfrac{3}{n}\right)^{2n}$

   j. $\lim\limits_{x->\infty} \dfrac{x^2}{\ln x}$

   k. $\lim\limits_{x->\infty} \left(e^{3x}+1\right)^{\frac{1}{2x}}$

   l. $\lim\limits_{x->0} \dfrac{\arctan(4x)}{x}$

2. Determine if each integral is improper. If it is improper, state why, re-write it using proper limit notation, and solve.

   a. $\int_0^{27} x^{-2/3}$

   b. $\int_0^{4} \dfrac{1}{\sqrt{4-x}}$

   c. $\int_1^{9} (x-1)^{-2/3}\,dx$

   d. $\int_0^{4} \dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx$

e. $\int_0^1 \frac{1}{e^x}\,dx$

f. $\int_2^6 \frac{1}{\sqrt{x-2}}\,dx$

g. $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

h. $\int_2^5 (x-1)^{-1/2}\,dx$

3. The series $4-3+\frac{9}{4}-\frac{27}{16}+\ldots$ is a geometric series. Find the general term, $a_n$, and write the sum in sigma notation. Does this series converge? If so, what is the sum?

4. Find the sum of the following (if possible):

a. $\sum_{k=0}^{\infty}\left(-\frac{3}{4}\right)^k$

b. $\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k$

c. $\sum_{k=0}^{\infty}\left(\frac{5}{4}\right)^{k-1}$

d. $\sum_{n=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

e. $\sum_{k=0}^{\infty}\frac{6^{k+1}}{7^{k-2}}$

5. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.

a. $\sum \frac{k^2 2^k}{(k+1)!}$

b. $\sum \frac{3^{k+1}}{(k+1)^2 e^k}$

c. $\sum \frac{\ln n}{n}$

d. $\sum \frac{2n+1}{\sqrt{n^5+3n^3+1}}$

e. $\sum \frac{4n^2+1}{n^3-n}$

f. $\sum \frac{4n^2+1}{n^5-n}$

g. $\sum \left(1+\frac{1}{n}\right)^n$

h. $\sum \frac{n^3}{3^n}$

i. $\sum \frac{1}{\sqrt[4]{n^3}}$

6. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

   a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$

   b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$

   c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$

   d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$

   e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$

7. Use the Taylor series expansion (in powers of x) for $f(x)=e^x$ to find the Taylor series expansion $f(x)=\cosh x$.

8. Determine the Taylor polynomial in powers of $x$ of degree 8 for the function $f(x)=x-\cos(x^2)$.

9. Determine the Taylor polynomial in powers of $x$ of degree 5 for the function $f(x)=\frac{1-e^x}{x}$

10. Determine the Taylor polynomial in powers of $x$- $\pi$ of degree 4 for the function $f(x)=\sin(2x)$.

11. Assume that $f$ is a function such that $|f^{(n)}(x)|\le 2$ for all $n$ and $x$.

    a. Estimate the maximum possible error if $P_4(0.5)$ is used to approximate $f(0.5)$

    b. Find the least integer $n$ for which $P_n(0.5)$ approximates $f(0.5)$ with an error less than $10^{-3}$.

12. Use the values in the table below and the formula for Taylor polynomials to give the 5th degree Taylor polynomial for $f$ centered at $x = 0$.

| $f(0)$ | $f'(0)$ | $f''(0)$ | $f'''(0)$ | $f^{(4)}(0)$ | $f^{(5)}(0)$ |
|---|---|---|---|---|---|
| 1 | 0 | -2 | 3 | -4 | 1 |