Math 1432
Exam 2 Review

1. Determine if the following are one-to-one, if so, find $f^{-1}(x)$
   a. $f(x) = x^3 + 1$
   b. $f(x) = 3x + 10$
   c. $f(x) = \sqrt{9 - x^2}$

2. Suppose f has an inverse, $f(3) = 1$ and $f'(3) = \dfrac{2}{7}$. Find $(f^{-1})'(1)$.

3. Suppose $f(x)$ is an invertible differentiable function and the graph of f passes through the points $(6, -1)$ and $(-1, 2)$. The slope of the tangent line to the graph of f at $x = -1$ is 7/2. Find the equation of the tangent line to the inverse of f at 2.

4. Find $(f^{-1})'(a)$ if $f(x) = x^3 + 1$ and $a = 9$

5. Find the derivative:
   a. $y = \ln\sqrt{e^x + 4x}$
   b. $y = \sin\left(\ln(5 - x)^6\right)$
   c. $y = x^2 e^{2x} + \ln e^{2x}$
   d. $y = e^{x^2} \cdot \cosh(3x)$
   e. $f(x) = \ln\left(\sec\sqrt{x}\right)$
   f. $f(x) = \dfrac{e^{\sqrt{x}}}{x^3}$
   g. $y = (\cos x)^{(x+7)}$
   h. $f(x) = (3x - 1)^{2x+6}$
   i. $f(x) = \ln(5x^2) + e^{6x} + \arctan(5 - 2x)$
   j. $f(x) = \log_7\left(3x^2\right)$
   k. $y = 6^{-2x}$
   l. $f(x) = \arctan(2x^3)$

6. Integrate:
   a. $\displaystyle\int_e^{4e} \frac{1}{x}\,dx$
   b. $\displaystyle\int\left(\frac{\csc^2 x}{2 + 5\cot x} - e^{9x}\right)dx$
   c. $\displaystyle\int\frac{\sinh x}{(2 + \cosh x)^2}\,dx$

d. $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}} dx$

e. $\int \dfrac{2}{\sqrt{x}\left(3-\sqrt{x}\right)} dx$

f. $\int \dfrac{x+2}{x+1} dx$

g. $\int \dfrac{3x^2+3x+3}{x^2+1} dx$

h. $\int \dfrac{\cos^3 x - \sin^2 x}{\cos^2 x} dx$

i. $\int \tan(3x) dx$

j. $\int \dfrac{\arctan(3x)}{1+9x^2} dx$

k. $\displaystyle\int_{0}^{\frac{\sqrt{3}}{2}} \dfrac{1}{\sqrt{1-x^2}} dx$

l. $\int \cos^4 x \sin^3 x\, dx$

m. $\int \cos^5 x \sin^2 x\, dx$

n. $\int \cot^3 x\, dx$

o. $\int x \ln(2x) dx$

p. $\int 2x \sin(3x) dx$

q. $\int \dfrac{5}{36+(x-1)^2} dx$

r. $\int \tan^4(x) dx$

s. $\int 2x \sec\left(4x^2\right) dx$

t. $\int \sec^4(x) dx$

7. Give the general solution for $\dfrac{dy}{dx} = (y+5)(x+2)$

8. Find the specific solution given the initial condition: $\dfrac{dy}{dx} = y-2 \quad y(0) = 6$

9. The number N of bacteria in a culture is given by $N = 200e^{kt}$. If N = 300 when t = 4 hours, find k (to the nearest tenth) and then determine approximately how long it will take for the number of bacteria to triple in size.

10. Suppose that the population of Zeegers grows at a rate proportional to itself, doubling every 12500 years. When the Zeeger population has reached 93 percent more than their current population, they plan to invade Earth. How many years will it be before the Zeegers attack Earth?

11. At what rate r of continuous compounding does a sum of money double in 15 years?

12. Give the equation for the tangent and normal to the curve: $f(x) = \ln(2x - 5) + e^{x-3}$ at the point (3,1).