Math 1432

Final Exam Review

1. Give the equation of the tangent line to the given graph at the point where x = 0
   a. $f(x) = \ln(6x+1) + e^{2x}$
   b. $f(x) = \ln(2x+1) - 3e^{-4x}$
   c. $f(x) = \sqrt{9-x^2}$
2. Find the inverse of the following:
   a. $f(x) = \dfrac{2}{3-x}$
   b. $f(x) = \dfrac{x+1}{x+2}$
3. Find the derivative of the inverse for the following:
   a. $f(x) = x^3 + 1, \quad f(2) = 9, \quad \left(f^{-1}\right)'(9) =$
   b. $f(-3) = 1,\ f(1) = 2,\ f'(-3) = 3,\ f'(1) = -2, \quad \left(f^{-1}\right)'(1) =$
   c. $f(x)$ passes through the points (3 , -2) and (-2 , 1). The slope of the tangent line to the graph of $f(x)$ at ***x*** = 3 is -1/4. Evaluate the derivative of the inverse of f at -2.
4. Find the equation of the tangent and the normal lines to the parametric curves at the given points:
   a. $x(t) = -2\cos 2t,\ y(t) = 4 + 2t, (-2,4)$
   b. $x(t) = 3\cos(3t) + 2t,\ y(t) = 1 + 5t, (3,1)$
5. Give an equation relating *x* and *y* for the curve given parametrically by
   a. $x(t) = -1 + 3\cos t \quad y(t) = 1 + 2\sin t$
   b. $x(t) = -1 + 3\cosh t \quad y(t) = 1 + 2\sinh t$
   c. $x(t) = -1 + 4e^{t} \quad y(t) = 2 + 3e^{-t}$
6. Differentiate the function:
   a. $f(x) = 3^{x^2}$
   b. $f(x) = \tan\left(\log_5 x\right)$
   c. $f(x) = x^{\sin x}$
   d. $f(x) = \sinh(3x)$
   e. $f(x) = \dfrac{\cosh x}{x}$

7. Integrate:
   a. $\int (\cosh(3x) + \sinh(2x))dx$
   b. $\int 4^{3x}\,dx$
   c. $\int \frac{\log_2(x^3)}{x}dx$
   d. $\int (2^{7x} - \sinh(5x))dx$
   e. $\int \frac{\sin(3x)}{16 + \cos^2(3x)}dx$
   f. $\int \frac{6x}{4 + x^4}dx$
   g. $\int \tan(3x)dx$
   h. $\int \frac{\arctan(3x)}{1 + 9x^2}dx$
   i. $\int \frac{1}{\sqrt{4 + x^2}}dx$
   j. $\int \sqrt{9 - x^2}\,dx$
   k. $\int 3\ln(4x)dx$
   l. $\int x^2 e^x dx$
   m. $\int \frac{5x + 14}{(x+1)(x^2 - 4)}dx$
   n. $\int \frac{x^2 + 5x + 2}{(x+1)(x^2 + 1)}dx$
   o. $\int \frac{2x^2}{\sqrt{9 - x^2}}dx$
   p. $\int 2\arctan(10x)dx$
   q. $\int 3x\cos(2x)dx$
8. Write an expression for the nth term of the sequence:
   a. 1, 4, 7, 10, ...
   b. 2, -1, $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$

9. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.
   a. $a_n = \dfrac{2n}{1+n}$
   b. $a_n = \dfrac{\cos n}{n}$
10. Determine if the following sequences converge or diverge. If they converge, give the limit.
   a. $\left\{(-1)^n\left(\dfrac{n}{n+1}\right)\right\}$
   b. $\left\{\dfrac{6n^2-2n+1}{4n^2-1}\right\}$
   c. $\left\{\dfrac{(n+2)!}{n!}\right\}$
   d. $\left\{\dfrac{3}{e^n}\right\}$
   e. $\left\{\dfrac{4n+1}{n^2-3n}\right\}$
   f. $\left\{\dfrac{e^n}{n^3}\right\}$
11. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.
   a. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{n+3}$
   b. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos \pi n}{n^2}$
   c. $\displaystyle\sum_{n=0}^{\infty}\frac{4n(-1)^n}{3n^2+2n+1}$
   d. $\displaystyle\sum_{n=0}^{\infty}\frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$
   e. $\displaystyle\sum_{n=0}^{\infty}\frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$
   f. $\displaystyle\sum_{n=0}^{\infty}\left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$

g. $\sum_{n=0}^{\infty}\left(\frac{2(-1)^n \arctan n}{3+n^2+n^3}\right)$

h. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3^n}{4^n+3n}\right)$

i. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3}{(n+2)\ln(n+2)}\right)$

12. Find the sum of the following convergent series:

a. $\sum_{n=0}^{\infty} 2\left(-\frac{4}{9}\right)^n$

b. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{5}{6^n}\right)$

13. State the indeterminate form and compute the following limits :

a. $\lim_{n\to\infty}\frac{\ln(n+4)}{n+2}$

b. $\lim_{n\to\infty}(3n)^{\frac{2}{n}}$

c. $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}$

d. $\lim_{n\to 0}\frac{x-\sin(2x)}{x+\sin(2x)}$

e. $\lim_{x->0}\frac{e^{x^2}-1}{2x^2}$

f. $\lim_{x->0+}\left(\frac{1}{x}\right)^x$

g. $\lim_{x->0}\frac{3e^{x/3}-(3+x)}{x^2}$

h. $\lim_{x->\infty}\frac{x^2}{\ln x}$

i. $\lim_{x->0}\frac{1+x-e^x}{x(e^x-1)}$

j. $\lim_{x->0}\frac{\arctan(4x)}{x}$

14. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$

b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$

15. For each of the problems in number 14, give the antiderivate F of the power series so that F(0)=0.

16. Evaluate each improper integral:

a. $\int_0^{27} x^{-2/3}$

b. $\int_0^4 \frac{1}{\sqrt{4-x}}$

17. Find the formula for the area of $r = 1 + 2\sin\theta$

a. Inside inner loop
b. Inside outer loop but outside inner loop
c. Inside outer loop and below x-axis

18. Find the smallest value of n so that the nth degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at x = 0 approximates $\ln(2)$ with an error of no more than 0.001 (also be able to do this with some of the other Taylor Polynomials)

19. Find the radius of convergence and interval of convergence for the following Power series:

a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$

b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$

d. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n n!}{n^n}$

20. Use logarithmic differentiation to find the derivative of:

a. $y = (3x-1)^{\sin(x)}$

b. $y = (x+1)^{\ln(x)}$

c. $y = (x^2+2)^{(\frac{1}{\ln x})}$

21. Determine the convergence or divergence for each series with the given general term:

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$		
$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n} \right)$		
$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$		
$\sum_{n=1}^{\infty} \cos(\pi n)$		
$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$		
$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}$		
$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2} \right)^n$		
$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$		
$\sum_{n=1}^{\infty} n e^{-n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$		
$\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$		
$\sum_{n=0}^{\infty} \left(\frac{2}{9} \right)^n$		
$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$		

$\sum_{n=1}^{\infty}(0.34)^n$		
$\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$		
$\sum_{n=1}^{\infty}\frac{1}{2n+1}$		