NAME: Sol
PS ID:

MATH 1432 - QUIZ 9
August 6, 2014

Show your work to get proper credit.

(1)[4 Pts] Evaluate each improper integral, and explain why it is improper:
(a)

$$\int_{-1}^{2} \frac{1}{x^2}dx = \int_{-1}^{0} \frac{dx}{x^2} + \int_{0}^{2} \frac{dx}{x^2} = \lim_{a\to 0}\left[\int_{-1}^{a}\frac{dx}{x^2} + \int_{a}^{2}\frac{dx}{x^2}\right]$$

$$= \lim_{a\to 0}\left[\underbrace{\left(-\frac{1}{x}\right)\Big|_{-1}^{a}}_{\text{Diverges}} + \left(-\frac{1}{x}\right)_a^2\right] \Rightarrow \boxed{\text{Diverges}}$$

(b)

$$\int_0^4 \frac{1}{\sqrt{4-x}}dx = \int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{a\to 4}\left[\int_0^a \underbrace{\frac{dx}{\sqrt{4-x}}}_{(4-x)^{-\frac{1}{2}}}\right] = \lim_{a\to 4}\left[-2(4-x)^{\frac{1}{2}}\right]_0^a$$

$$= \lim_{a\to 4}\left[-2\sqrt{4-a}+2\sqrt{4}\right] = \boxed{4}$$

(2)[4 Pts] Does the following series converge or diverge? If it converges, what does it converge to?

$$\sum_{n=3}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right) = \underset{n=3}{\left(\frac{1}{5}-\cancel{\frac{1}{7}}\right)} + \underset{n=4}{\left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{9}}\right)} + \underset{n=5}{\left(\cancel{\frac{1}{9}}-\cancel{\frac{1}{11}}\right)} + \underset{n=6}{\left(\cancel{\frac{1}{11}}-\cancel{\frac{1}{13}}\right)} + \cdots$$

$$= \frac{1}{5} \text{ which converges}$$

(3)[2 Pts] Find the following sums (note that these are not 'infinite sums'):
(a)

$$\sum_{k=0}^{4}\frac{8}{2^k} = \frac{8}{2^0}+\frac{8}{2^1}+\frac{8}{2^2}+\frac{8}{2^3}+\frac{8}{2^4} = 8+4+2+1+\frac{1}{2} = \boxed{15\frac{1}{2} \text{ or } \frac{31}{2}}$$

(b)

$$\sum_{k=0}^{3}\frac{7}{3^k} = \frac{7}{3^0}+\frac{7}{3^1}+\frac{7}{3^2}+\frac{7}{3^3} = 7\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}\right)$$

$$= 7\cdot\left(\frac{27+9+3+1}{27}\right) = 7\cdot\frac{40}{27} = \boxed{\frac{280}{27}}$$