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Show your work to get proper credit.

(1) [3 Pts] Calculate the limit of the following sequences:

(a) $a_n = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n \rightarrow 0$ as $n \rightarrow \infty$ (This is an exponential function, not a polynomial)

(b) $a_n = \frac{5n - 3}{1 - n^2} = \frac{P(n)}{Q(n)}$, $\deg(P) = 1 < 2 = \deg(Q) \Rightarrow a_n \rightarrow 0$ as $n \rightarrow \infty$

(c) $a_n = \frac{254n^2 + 4n^3 - 17}{n^3 + 435n^2 + 12} = \frac{P(n)}{Q(n)}$, $\deg(P) = 3 = \deg(Q)$
 $\Rightarrow a_n \xrightarrow[\text{coefficient}]{\text{leading}} \frac{4}{1} = 4$ as $n \rightarrow \infty$

(2) [4 Pts] Give the LUB and GLB for the following sequence

$a_n = \frac{7n + (-1)^n}{n}$ $n = 1, 2, 3, \dots$

$= \frac{7n}{n} + \frac{(-1)^n}{n}$
 $= 7 + \frac{(-1)^n}{n}$

$a_1 = 6$

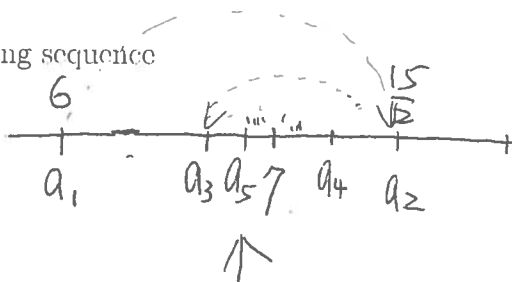
$a_2 = 7 + \frac{1}{2} = \frac{15}{2}$

$a_3 = 7 - \frac{1}{3}$

$a_4 = 7 + \frac{1}{4}$

$a_5 = 7 - \frac{1}{5}$

$a_6 = 7 + \frac{1}{6}$



see this number line.

we have

LUB = $\frac{15}{2}$
 GLB = 6.

(3) [3 Pts] Write the first three terms of the following sequence:

$a_1 = 2, \quad a_{n+1} = (2a_n + 3)^2, \quad \text{for } n = 1, 2, 3, \dots$

$a_1 = 2$
 $a_2 = a_{1+1} = (2a_1 + 3)^2 = (2 \times 2 + 3)^2 = 7^2 = 49$
 $a_3 = a_{2+1} = (2a_2 + 3)^2 = (2 \times 49 + 3)^2 = (101)^2 = 10201$

$a_1 = 2$
 $a_2 = 49$
 $a_3 = 10201$