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MATH 1432 - QUIZ 7

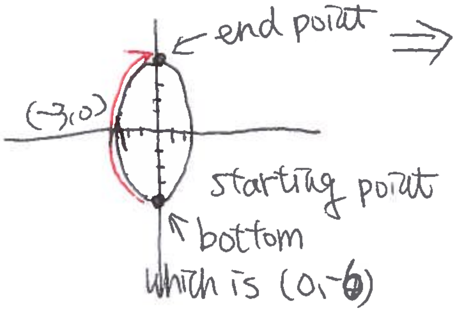
July 31, 2014

Show your work to get proper credit.

(1)[3 Pts] Give a parameterization for a particle moving along the part of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{36} = 1 \Rightarrow \left(\frac{x}{3}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$

that starts at the bottom of the ellipse, goes through the point $(-3, 0)$ and stops at the top of the ellipse.



$\Rightarrow$ let $t \in [0,1]$. Starting point is $(0,-6)$, go clockwise for half ellipse

period $= \frac{2\pi}{1-0} \cdot \frac{1}{2} = \pi$

let $x(t) = 3\cos(-\pi t + \theta_0)$, $y(t) = 6\sin(-\pi t + \theta_0)$ $\Rightarrow$ At $t=0$: $x(0) = 0$, $y(0) = -6$

$\Rightarrow \cos(\theta_0) = 0$, $\sin(\theta_0) = -1$ $\Rightarrow \theta_0 = \frac{3\pi}{2}$

$$\Rightarrow \boxed{\begin{aligned} x(t) &= 3\cos(-\pi t + \tfrac{3\pi}{2}) \\ y(t) &= 6\sin(-\pi t + \tfrac{3\pi}{2}) \end{aligned}}$$

or $= 3\cos\frac{3\pi}{2}\cos\pi t + 3\sin\frac{3\pi}{2}\sin\pi t = \boxed{-3\sin\pi t}$

$= 6\sin\frac{3\pi}{2}\cos\pi t - 6\cos\frac{3\pi}{2}\sin\pi t = \boxed{-6\cos\pi t}$

$t \in [0,1]$

(2)[3 Pts] Give an integral which represents the length of the curve given parametrically by

$$r(t) = (-8\cos(t), 4\sin(t)) \quad \text{for} \quad 0 \le t \le \frac{3}{2}\pi.$$
$= (x(t), y(t))$

$x = -8\cos(t)$, $y = 4\sin(t)$ $\Rightarrow$ $x' = +8\sin(t)$, $y' = 4\cos(t)$.

The length between $t \in [0, \frac{3\pi}{2}]$ is $\int_0^{\frac{3\pi}{2}} \sqrt{[x']^2 + [y']^2}\, dt$

$$= \int_0^{\frac{3\pi}{2}} \sqrt{(8\sin(t))^2 + (4\cos(t))^2}\, dt.$$

(3)[4 Pts] Find an equation of the tangent line to the curve at the given value:

$$x(t) = -8\cos(t), \qquad y(t) = 4e^{2t}\sin(t) \qquad t = \frac{3}{2}\pi$$

$(y + 4e^{3\pi}) = e^{3\pi} x.$

Point: $\left(x(\frac{3\pi}{2}), y(\frac{3\pi}{2})\right) = (0, -4e^{3\pi})$

Slope: $\left.\frac{dy}{dx}\right|_{t=\frac{3\pi}{2}} = \left.\frac{y'(t)}{x'(t)}\right|_{t=\frac{3\pi}{2}} = \left.\frac{8e^{2t}\sin(t) + 4e^{2t}\cos(t)}{8\sin(t)}\right|_{t=\frac{3\pi}{2}} = \frac{8e^{3\pi}(-1) + 4e^{3\pi}\cdot 0}{8\cdot(-1)} = e^{3\pi}$