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MATH 1432 - QUIZ 6

July 28, 2014

Show your work to get proper credit.

5 (1) [~~3~~ Pts] Determine the value of n which guarantees a theoretical error less than $\varepsilon = 0.001$ when estimating the following integral by Simpson's rule. For this problem only give the inequality that helps you find n (since you don't have a calculator).

$$\int_2^5 \frac{1}{x}dx \quad f(x)=\frac{1}{x},\ f'(x)=-x^{-2},\ f''(x)=+2x^{-3},\ f'''(x)=-6x^{-4},\ f^{(4)}(x)=24x^{-5}$$

$$x\in[2,5] \Rightarrow \max_{x\in[2,5]}|f^{(4)}(x)| = 24\cdot(2)^{-5}=\frac{24}{32}.$$

$$\Rightarrow |E_n^S| = \left|-\frac{(5-2)^5}{2880\,n^4}\cdot f^{(4)}(c)\right| \le \left|\frac{3^5}{2880\,n^4}\cdot\frac{24}{32}\right| \le 0.001 = \frac{1}{1000}$$

$c\in[2,5]$

$$\Rightarrow \frac{(3)^5\cdot 24\cdot 1000}{2880\cdot 32} \le n^4$$

5 (2) [~~3~~ Pts] Determine the value of n which guarantees a theoretical error less than $\varepsilon = 0.001$ when estimating the following integral by the trapezoidal rule. For this problem only give the inequality that helps you find n (since you don't have a calculator).

$$\int_3^{e^2} \ln(x)\,dx \quad f(x)=\ln(x),\ f'(x)=\frac{1}{x},\ f''(x)=-x^{-2},\ \Rightarrow \max_{x\in[3,e^2]}|f''(x)|=\frac{1}{9}$$

$x\in[3,e^2]$

$$\Rightarrow |E_n^T| = \left|-\frac{(e^2-3)^3}{12n^2}\cdot f''(c)\right| \le \frac{(e^2-3)^3}{12n^2}\cdot\frac{1}{9} \le 0.001 = \frac{1}{1000}$$

$c\in[3,e^2]$

$$\Rightarrow \frac{1000\,(e^2-3)^3}{108} \le n^2$$

(3) [~~4~~ Pts] Identify the curve and write the equation in rectangular coordinates:

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$$r = 7(1-\cos\theta)^{-1}$$

(Bonus). let $r^2=x^2+y^2$, $\cos\theta=\frac{x}{r}$

Then we have $r=\frac{7}{1-\cos\theta} \Leftrightarrow r(1-\cos\theta)=7$

$\Leftrightarrow r(1-\frac{x}{r})=7 \Leftrightarrow r-x=7 \Leftrightarrow r=7+x \Leftrightarrow \sqrt{x^2+y^2}=7+x$

Take square $\Leftrightarrow x^2+y^2=(7+x)^2=49+14x+x^2 \Leftrightarrow y^2=14x+49$ which is parabola