NAME: Sol

PS ID:

MATH 1432 - QUIZ 5

July 24, 2014

Show your work to get proper credit.

(1)[3 Pts] Use long division to rewrite the improper rational function as a sum of a polynomial and a proper rational function.

$$\frac{x^5+2}{x^2-1}$$

$$\begin{array}{r} x^3+0x^2+x+0 \quad Q(x) \\ x^2+0x-1 \,\big)\, \overline{x^5+0x^4+0x^3+0x^2+0x+2} \\ -\left(x^5+0x^4-x^3\right) \\ \hline 0x^4+x^3+0x^2 \\ -\left(0x^4+0x^3-0x^2\right) \\ \hline x^3+0x^2+0x \\ -\left(x^3+0x^2-x\right) \\ \hline +x+2 \quad R(x) \end{array}$$

$$\frac{x^5+2}{x^2-1} = x^3+x+\frac{x+2}{x^2-1}$$

(2)[3 Pts] Calculate the following integral, and simplify using properties of the natural logarithm.

$$\int \frac{2x}{x^2-x-2}dx = \int \frac{2x}{(x-2)(x+1)}dx = \int \frac{4}{3}\cdot\frac{1}{x-2}dx + \int \frac{2}{3}\cdot\frac{1}{x+1}dx$$

$$= \frac{4}{3}\ln|x-2| + \frac{2}{3}\ln|x+1| + C$$

let $f(x) = \dfrac{2x}{(x-2)(x+1)} = \dfrac{A}{x-2} + \dfrac{B}{x+1}$

$A = \dfrac{4}{3}$, $B = \dfrac{2}{3}$

Q3. Let $x = \frac{3}{2}\tan\theta \Rightarrow \tan\theta = \frac{2x}{3}$

$\Rightarrow dx = \frac{3}{2}\sec^2\theta\, d\theta$, $\sqrt{4x^2+9} = \sqrt{4\cdot\frac{9}{4}\tan^2\theta+9}$

$= \sqrt{9+9\tan^2\theta} = 3\sec\theta$

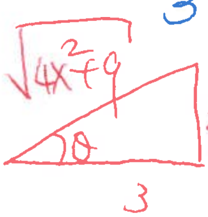
$\left(\sec\theta = \frac{1}{\cos\theta},\ \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

We have

(3)[4 Pts] Calculate the following integral

$$\int \frac{1}{x\sqrt{4x^2+9}}dx = \int \frac{\frac{3}{2}\sec^2\theta\, d\theta}{\frac{3}{2}\tan\theta\cdot 3\sec\theta} = \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}d\theta = \frac{1}{3}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}d\theta$$

$$= \frac{1}{3}\int \frac{d\theta}{\sin\theta} = \frac{1}{3}\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$$



$\tan\theta = \frac{2x}{3}$, $\csc\theta = \frac{\sqrt{4x^2+9}}{2x}$, $\cot\theta = \frac{3}{2x}$

$$= \ln\left|\frac{\sqrt{4x^2+9}}{2x} - \frac{3}{2x}\right| + C$$

$$\int \csc\theta \, d\theta = \int \csc\theta \cdot \left(\frac{\csc\theta - \cot\theta}{\csc\theta - \cot\theta} \right) d\theta$$

$$= \int \frac{\csc^2\theta - \csc\theta\cot\theta}{\csc\theta - \cot\theta} \, d\theta$$

let $u = \csc\theta - \cot\theta$, $du = -\csc\theta\cot\theta - (-\csc^2\theta) \, d\theta$

$$= \csc^2\theta - \csc\theta\cot\theta$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\csc\theta - \cot\theta| + C$$